



UPSC PHYSICS OPTIONAL for CSE & IFoS

ABHI PHYSICS

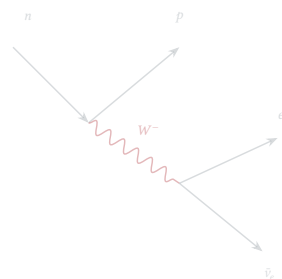
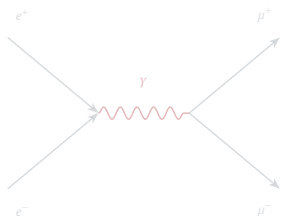
$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$$

$$[\hat{x}, \hat{p}] = i\hbar$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

PREVIOUS YEAR QUESTIONS



41 YEARS QUESTION BANK | 1986-2026

CSE + IFoS

SUBJECT

Electrodynamics

10 SOLVED QUESTIONS • CSE + IFoS

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$E^2 = p^2 c^2 + m^2 c^4$$

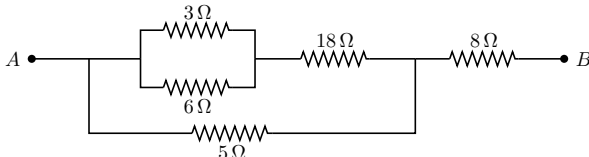
$$\partial_\mu j^\mu = 0$$

SUBJECT-WISE • TOPIC-WISE • SOURCE-TAGGED

For Civil Services and Indian Forest Service Examinations

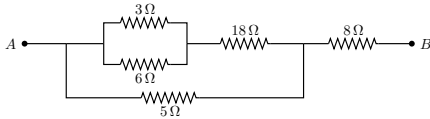
Sample Preview extbar 10 Solved Questions

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7	Consider a point charge of 5 nC placed at a distance of 1 m from a perfect conducting plane ($z = 0$) of infinite extent. Find the electric field at a point $(2, 2, 0) \text{ m}$ and show that it is normal to the plane.	10	2025	CSE	10
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9	State and explain Kirchhoff's current law and Kirchhoff's voltage law. Derive these laws from the principles of charge conservation and energy conservation.	10	2025	CSE	12
10	A parallel plate capacitor having circular plates of radius 10 cm is being charged. If the electric field at any instant within the capacitor changes at the rate $5 \cdot 0 \text{ V m}^{-1} \text{ s}^{-1}$, calculate the magnetic intensity $ \vec{H} $ inside the capacitor.	10	2025	CSE	15

CSE • 2026 • 10 MARKS

Q1. Calculate the effective resistance of the following combination of resistances as shown in the figure and determine the voltage drop across each resistance when a potential difference of 120 volts is applied between points A and B :

**INTRODUCTION:**

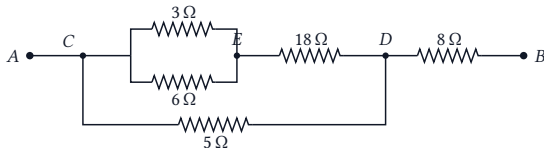
The circuit is to be reduced by identifying resistors connected between the same pair of nodes and then applying the series and parallel combination rules. All resistances are assumed ideal, and the connecting wires have negligible resistance. Let

$$V_A - V_B = 120 \text{ V},$$

so conventional current is taken from A toward B . Denote by C the node immediately after A , by D the junction before the 8Ω resistor, and by E the junction between the 3Ω - 6Ω parallel pair and the 18Ω resistor. Ohm's law and Kirchhoff's current and voltage laws will be used.

SOLUTION:

The circuit topology is



The 3Ω and 6Ω resistors are connected between the same nodes C and E , and hence are in parallel:

$$R_{CE} = \frac{(3 \Omega)(6 \Omega)}{3 \Omega + 6 \Omega} = \frac{18}{9} \Omega = 2 \Omega.$$

This 2Ω equivalent is in series with the 18Ω resistor along the upper path from C to D . Therefore,

$$R_{\text{upper}} = 2 \Omega + 18 \Omega = 20 \Omega.$$

The upper 20Ω path and the lower 5Ω path have the same end nodes C and D , so they are in parallel:

$$R_{CD} = \frac{(20 \Omega)(5 \Omega)}{20 \Omega + 5 \Omega} = \frac{100}{25} \Omega = 4 \Omega.$$

Finally, R_{CD} is in series with the 8Ω resistor:

$$R_{AB} = R_{CD} + 8 \Omega = 4 \Omega + 8 \Omega = 12 \Omega.$$

The total current supplied between A and B is consequently

$$I = \frac{V_A - V_B}{R_{AB}} = \frac{120 \text{ V}}{12 \Omega} = 10 \text{ A}.$$

Since the 8Ω resistor is in series with the entire C - D network, it carries 10 A . Its voltage drop from D toward B is

$$V_8 = I(8 \Omega) = (10 \text{ A})(8 \Omega) = 80 \text{ V}.$$

Kirchhoff's voltage law then gives the voltage across the parallel network between C and D :

$$V_{CD} = 120 \text{ V} - 80 \text{ V} = 40 \text{ V}.$$

The 5Ω resistor is directly connected between C and D , so

$$V_5 = V_{CD} = 40 \text{ V}, \quad I_5 = \frac{40 \text{ V}}{5 \Omega} = 8 \text{ A}.$$

The complete upper path also has a 40 V drop. Its current is

$$I_{\text{upper}} = \frac{40 \text{ V}}{20 \Omega} = 2 \text{ A}.$$

Thus, the drop across the 18Ω resistor is

$$V_{18} = I_{\text{upper}}(18 \Omega) = (2 \text{ A})(18 \Omega) = 36 \text{ V}.$$

The remaining voltage across the 3Ω - 6Ω parallel pair is

$$V_{CE} = V_{CD} - V_{18} = 40 \text{ V} - 36 \text{ V} = 4 \text{ V}.$$

Since both resistors are connected between C and E ,

$$V_3 = V_6 = 4 \text{ V}.$$

Their respective currents are

$$I_3 = \frac{4 \text{ V}}{3 \Omega} = \frac{4}{3} \text{ A}, \quad I_6 = \frac{4 \text{ V}}{6 \Omega} = \frac{2}{3} \text{ A}.$$

These add to

$$I_3 + I_6 = \frac{4}{3} \text{ A} + \frac{2}{3} \text{ A} = 2 \text{ A},$$

which is the current entering the pair. At node C ,

$$I_5 + I_{\text{upper}} = 8 \text{ A} + 2 \text{ A} = 10 \text{ A},$$

in agreement with the total circuit current. The voltage drops along the upper path also satisfy

$$V_{3\parallel 6} + V_{18} = 4 \text{ V} + 36 \text{ V} = 40 \text{ V},$$

and the complete A -to- B drop is

$$40 \text{ V} + 80 \text{ V} = 120 \text{ V}.$$

CONCLUSION:

The effective resistance between A and B is

$$R_{AB} = 12 \Omega,$$

and the total conventional current is

$$I = 10 \text{ A}$$

from A toward B . The voltage decreases in the direction of this current, with the individual drops

$$V_{3\Omega} = 4 \text{ V}, \quad V_{6\Omega} = 4 \text{ V}, \quad V_{18\Omega} = 36 \text{ V}, \quad V_{5\Omega} = 40 \text{ V}, \quad V_{8\Omega} = 80 \text{ V}.$$

These results apply for ideal ohmic resistors and negligible connecting-wire resistance. The equal 4 V drops across the 3Ω and 6Ω resistors follow from their parallel connection, while the 5Ω resistor has the same 40 V drop as the entire alternative upper branch.

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Q2. A charge of $-3.30 \mu\text{C}$ is fixed at a point. From a horizontal distance of 0.0455 m , a charged particle of mass $7.35 \times 10^{-3} \text{ kg}$ and charge $-7.45 \mu\text{C}$ is fired with an initial velocity 62.5 m/s directly towards the fixed charge. How far does this charge travel before its speed becomes zero?

INTRODUCTION:

The required quantity is the distance travelled by the moving charge before it reaches its point of closest approach and momentarily comes to rest. Since both charges are negative, their electrostatic interaction is repulsive. The particle is therefore decelerated as it moves towards the fixed charge.

Let the fixed charge be at $x = 0$, and let the moving particle initially be at $x = r_i > 0$. The positive x -direction is taken from the fixed charge towards the particle. Thus the initial velocity is directed along $-x$, whereas the electrostatic force is along $+x$.

Write

$$Q = -3.30 \times 10^{-6} \text{ C}, \quad q = -7.45 \times 10^{-6} \text{ C},$$

$$m = 7.35 \times 10^{-3} \text{ kg}$$

$$r_i = 0.0455 \text{ m}$$

$$v_i = 62.5 \text{ m s}^{-1}.$$

The fixed charge is assumed to remain stationary, both charges are treated as point charges, and forces other than the electrostatic force are neglected. Since the electrostatic force is conservative, mechanical energy is conserved.

SOLUTION:

For two point charges separated by a distance r , the electrostatic potential energy is

$$U(r) = \frac{kQq}{r}, \quad k = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}.$$

Here

$$Qq = (-3.30 \times 10^{-6})(-7.45 \times 10^{-6}) = 2.4585 \times 10^{-11} \text{ C}^2 > 0,$$

so $U(r)$ increases as the separation decreases. The particle's initial kinetic energy is converted into electrostatic potential energy.

Let r_t denote the separation at the turning point, where the speed becomes zero. Conservation of mechanical energy gives

$$\frac{1}{2}mv_i^2 + \frac{kQq}{r_i} = 0 + \frac{kQq}{r_t}.$$

Hence

$$\frac{1}{r_t} = \frac{1}{r_i} + \frac{mv_i^2}{2kQq}.$$

The initial kinetic energy is

$$\begin{aligned} K_i &= \frac{1}{2}(7.35 \times 10^{-3})(62.5)^2 \\ &= \frac{1}{2}(7.35 \times 10^{-3})(3906.25) \\ &= 14.36 \text{ J}. \end{aligned}$$

Also,

$$\begin{aligned} kQq &= (8.99 \times 10^9)(2.4585 \times 10^{-11}) \\ &= 2.21 \times 10^{-1} \text{ J m}. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{1}{r_t} &= \frac{1}{0.0455} + \frac{14.36}{2.21 \times 10^{-1}} \\ &= 21.98 \text{ m}^{-1} + 64.98 \text{ m}^{-1} \\ &= 86.96 \text{ m}^{-1}, \end{aligned}$$

and hence

$$r_t = \frac{1}{86.96} = 1.15 \times 10^{-2} \text{ m}.$$

The distance travelled towards the fixed charge is the decrease in separation:

$$\begin{aligned} s &= r_i - r_t \\ &= 0.0455 - 0.0115 \\ &= 0.0340 \text{ m}. \end{aligned}$$

Since $r_t < r_i$, the result correctly describes motion towards the fixed charge. At r_t , the positive electrostatic potential energy has increased by exactly the initial kinetic energy, so the particle momentarily stops and then reverses direction under the repulsive force.

CONCLUSION:

The moving charge travels

$$s = 3.40 \times 10^{-2} \text{ m}$$

or

$$s = 3.40 \text{ cm}$$

towards the fixed negative charge before its speed becomes zero. Its minimum separation from the fixed charge is

$$r_t = 1.15 \times 10^{-2} \text{ m} = 1.15 \text{ cm}.$$

This result applies under the point-charge approximation with the fixed charge stationary and all non-electrostatic forces neglected.

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Q3. Discuss the origin of hysteresis in ferromagnetic materials on the basis of domain theory. Explain how domain wall motion and pinning lead to energy loss during cyclic magnetization.

INTRODUCTION:

A ferromagnetic specimen below its Curie temperature is divided into magnetic domains. Within each domain, the atomic magnetic moments are aligned approximately parallel by the exchange interaction, so that the domain possesses a spontaneous magnetization M_s . Adjacent domains are separated by transition regions called domain walls, within which the magnetization rotates continuously from the direction in one domain to that in the next.

The domain structure minimizes the total free energy, which contains exchange, magnetocrystalline anisotropy, magnetostatic, magnetoelastic, and domain-wall contributions. In zero applied field, differently oriented domains generally arrange themselves so that the net macroscopic magnetization is small, even though each domain is strongly magnetized.

Let the applied magnetic field H and the measured magnetization M be positive along a fixed axis. In SI units,

$$B = \mu_0(H + M).$$

The hysteresis loop arises because the domain configuration at a given H depends on the previous magnetic history. The principal microscopic cause of this history dependence is the irreversible displacement

of domain walls through an inhomogeneous material containing defects, internal stresses, grain boundaries, inclusions, dislocations, and spatial variations of magnetic anisotropy.

SOLUTION:

When a field H is applied, the magnetic contribution to the free-energy density of a domain whose magnetization makes an angle θ with the field is

$$u_H = -\mu_0 \mathbf{M} \cdot \mathbf{H} = -\mu_0 M_s H \cos \theta.$$

Thus, domains whose magnetization lies more nearly parallel to the field have lower energy. Magnetization can then increase by two principal processes:

1. Domain walls move so that domains favourably oriented with respect to the field grow at the expense of unfavourably oriented domains.
2. At larger fields, the magnetization within the remaining domains rotates towards the field direction. Eventually the specimen approaches magnetic saturation.

Consider a wall separating two domains whose magnetization components along the applied field are M_1 and M_2 . If the wall moves through a distance dx , converting an elemental volume $A dx$ from domain 1 to domain 2, the change in magnetic energy is

$$dU_H = -\mu_0 H(M_2 - M_1)A dx.$$

Hence the field exerts an effective pressure on the wall,

$$p_H = \mu_0 H(M_2 - M_1).$$

For a 180° wall separating domains with components $-M_s$ and $+M_s$,

$$p_H = 2\mu_0 M_s H.$$

The wall therefore tends to move in the direction that enlarges the domain magnetized parallel to the field.

In an ideal, perfectly homogeneous crystal, a wall could move comparatively freely and much of the response would be reversible. A real ferromagnet, however, has a non-uniform internal energy landscape. A defect can lower the wall energy when the wall intersects it, or an internal stress can locally alter the magnetoelastic and anisotropy energies. Such regions act as pinning centres.

Let $U_p(x)$ denote the position-dependent pinning energy of a wall. The effective wall energy may be represented schematically by

$$U_{\text{eff}}(x; H) = U_p(x) - \mu_0 H(M_2 - M_1)Ax.$$

The applied field tilts this energy landscape. For a small field, a pinned wall may deform or move slightly within a local energy minimum. Such motion is approximately reversible: when the field is removed, the wall returns nearly to its original position.

As H increases, the field pressure increases. At a critical depinning field H_p , the local minimum ceases to hold the wall. In force form, the depinning condition is approximately

$$\mu_0 H_p (M_2 - M_1) A \simeq \max \left| \frac{dU_p}{dx} \right|.$$

The wall then jumps abruptly to another stable position. This is an irreversible change because, after the jump, merely restoring the field to its previous value does not restore the former domain configuration. The wall can return only when the field is reversed sufficiently to overcome a generally different pinning barrier. A succession of such discontinuous jumps produces Barkhausen noise.

Starting from an initially demagnetized state, increasing H causes favourable domains to grow by wall motion. At higher fields, domain rotation becomes important, and sufficiently large H produces approximate saturation. When the field is subsequently reduced to zero, many walls do not retrace their earlier paths because they become trapped at pinning centres. Consequently, a finite magnetization remains:

$$M(H = 0) = M_r,$$

where M_r is the remanent magnetization.

A reverse field is required to move the pinned walls and reverse enough domains to make the net magnetization vanish. The magnitude of the reverse field for which $M = 0$ is the coercive field H_c . Further increase of the reverse field produces negative saturation. Repeating the process completes a closed hysteresis loop.

The work supplied by the external source per unit volume in a quasistatic magnetic cycle is

$$W_{\text{cycle}} = \oint H dB.$$

Using $B = \mu_0(H + M)$,

$$W_{\text{cycle}} = \mu_0 \oint H dH + \mu_0 \oint H dM.$$

Since H returns to its initial value,

$$\oint H dH = \oint d\left(\frac{H^2}{2}\right) = 0,$$

and therefore

$$W_{\text{loss}} = \mu_0 \oint H dM.$$

Thus the energy dissipated per unit volume in one complete cycle equals the area enclosed by the B - H loop,

$$W_{\text{loss}} = \oint H dB,$$

or equivalently μ_0 times the area enclosed by the M - H loop. Its unit is

$$[H dB] = \frac{\text{A}}{\text{m}} \text{T} = \frac{\text{J}}{\text{m}^3},$$

as required for an energy density.

Microscopically, the applied field stores energy while a domain wall is held against a pinning barrier. Upon depinning, the wall accelerates and jumps to another metastable position. The released energy is transferred to lattice vibrations, electronic excitations, microscopic eddy currents in conducting specimens, and ultimately heat. Since the reverse path encounters barriers in a different sequence and direction, the energy released during forward motion is not fully recovered. The magnetization consequently lags behind the applied field, producing hysteresis and a positive energy loss per cycle.

The character of the hysteresis loop is therefore closely related to the strength of pinning. Soft ferromagnetic materials contain relatively mobile domain walls and have small H_c , narrow loops, and low hysteresis loss. Hard ferromagnetic materials possess strong pinning and large anisotropy; they consequently have large coercivity, broad loops, and large remanence, making them suitable for permanent magnets.

CONCLUSION:

Ferromagnetic hysteresis originates from the existence of multiple metastable domain configurations separated by energy barriers. An applied field lowers the energy of domains aligned with it, causing domain-wall motion and, at larger fields, domain rotation. Structural defects, internal stresses, grain boundaries, inclusions, and anisotropy variations pin the walls. Walls depin only after a finite threshold field is reached and then move irreversibly to new positions, so the domain pattern on decreasing or reversing the field differs from that formed while increasing it.

This irreversible wall motion accounts for remanent magnetization M_r , coercive field H_c , and the lag of M behind H . The magnetic energy dissipated per unit volume per cycle is

$$W_{\text{loss}} = \oint H dB = \mu_0 \oint H dM,$$

equal to the enclosed B - H loop area, or to μ_0 times the enclosed M - H loop area. The lost magnetic work ultimately appears mainly as heat; stronger wall pinning generally produces a wider hysteresis loop and a larger cyclic energy loss.

CSE · 2026 · 15 MARKS

Q4. A wire of length 2 m is perpendicular to the X - Y plane. It is moved with velocity $\vec{V} = 2\hat{i} + 3\hat{j} + \hat{k}$ m/s through a region of uniform magnetic induction $\vec{B} = \hat{i} + 2\hat{j}$ Wb/m². Compute the potential difference induced between the ends of the wire.

INTRODUCTION:

The wire is perpendicular to the X - Y plane and is therefore parallel to the z -axis. Let its lower end be A , its upper end in the $+\hat{k}$ direction be C , and

$$\vec{L}_{AC} = 2\hat{k} \text{ m.}$$

The calculation is made in the laboratory frame, in which the magnetic field is uniform and the wire translates rigidly with velocity $\vec{V}$. The wire is assumed to be open-circuited. The magnetic Lorentz force $q(\vec{V} \times \vec{B})$ separates charges until an electrostatic potential difference is established between its ends.

With the stated orientation, the required terminal potential difference is

$$V_C - V_A = \int_A^C (\vec{V} \times \vec{B}) \cdot d\vec{l}.$$

SOLUTION:

The motional-force vector per unit charge is

$$\begin{aligned} \vec{V} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} \\ &= (3 \cdot 0 - 1 \cdot 2)\hat{i} - (2 \cdot 0 - 1 \cdot 1)\hat{j} + (2 \cdot 2 - 3 \cdot 1)\hat{k} \\ &= -2\hat{i} + \hat{j} + \hat{k} \text{ V m}^{-1}. \end{aligned}$$

Only the component of $\vec{V} \times \vec{B}$ parallel to the wire contributes to the terminal potential difference. Since $d\vec{l} = \hat{k} dz$,

$$\begin{aligned} V_C - V_A &= \int_0^2 (-2\hat{i} + \hat{j} + \hat{k}) \cdot \hat{k} dz \\ &= \int_0^2 1 dz \\ &= 2 \text{ V.} \end{aligned}$$

The $+\hat{k}$ component of $\vec{V} \times \vec{B}$ drives positive charges toward C . Thus the end in the $+\hat{k}$ direction is at the higher potential.

The result also follows from the scalar triple product

$$V_C - V_A = (\vec{V} \times \vec{B}) \cdot \vec{L}_{AC} = 2 \text{ V.}$$

This shows that the velocity component parallel to the wire does not contribute directly; the nonzero voltage arises from the transverse velocity and magnetic-field components whose cross product has a component along the wire.

CONCLUSION:

The magnitude of the induced potential difference between the ends of the wire is

$$\Delta V = 2 \text{ V.}$$

For the convention $\vec{L}_{AC} = 2\hat{k}$ m,

$$V_C - V_A = +2 \text{ V,}$$

so the end lying in the $+\hat{k}$ direction is 2 V above the end lying in the $-\hat{k}$ direction.

CSE · 2026 · 15 MARKS

Q5. A very long solenoid of radius a , with n turns per unit length, carries a current I_s . Coaxial with the solenoid, at radius $b \gg a$, is a circular ring of wire with resistance R . When the current in the solenoid is gradually reduced, a current I_r is induced in the ring. Calculate I_r in terms of $\frac{dI_s}{dt}$. Also, calculate the power dissipated through Joule effect and the electric field $\vec{E}$ near the solenoid.

INTRODUCTION:

For an ideal, very long solenoid, the magnetic field is uniform inside and negligible outside:

$$\vec{B} = \begin{cases} \mu_0 n I_s \hat{z}, & r < a, \\ 0, & r > a. \end{cases}$$

Although the ring has radius $b \gg a$, the magnetic flux linked with it comes only from the cross-sectional area πa^2 of the solenoid.

Choose $+\hat{z}$ along the magnetic field produced by positive I_s , and choose $+\hat{\phi}$ counterclockwise as viewed from $+\hat{z}$. Faraday's law,

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt},$$

determines both the induced emf and the circulating electric field. The current is assumed to vary quasistatically, displacement-current and radiation effects are neglected, and the ring's self-inductance is neglected compared with its resistance; equivalently, the variation time scale is much longer than its L/R time.

SOLUTION:

The magnetic flux through the ring is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = (\mu_0 n I_s)(\pi a^2).$$

Therefore, the induced emf around the ring is

$$\mathcal{E}_r = -\frac{d\Phi_B}{dt} = -\mu_0 n \pi a^2 \frac{dI_s}{dt}.$$

Using Ohm's law for the ring,

$$I_r = \frac{\mathcal{E}_r}{R},$$

gives

$$I_r = -\frac{\mu_0 n \pi a^2}{R} \frac{dI_s}{dt}.$$

Here $I_r > 0$ denotes current in the $+\hat{\phi}$ direction. Since the solenoid current is being reduced,

$$\frac{dI_s}{dt} < 0,$$

and hence $I_r > 0$: the induced current is counterclockwise when viewed from $+\hat{z}$. Its magnetic field points along $+\hat{z}$, opposing the decrease of the solenoid's flux, as required by Lenz's law.

The Joule power dissipated in the ring is

$$P_J = I_r^2 R.$$

Substitution yields

$$P_J = \frac{\mu_0^2 n^2 \pi^2 a^4}{R} \left(\frac{dI_s}{dt} \right)^2.$$

The square makes the dissipated power positive irrespective of whether I_s is increasing or decreasing.

To determine the electric field, symmetry requires it to be azimuthal:

$$\vec{E}(r) = E_\phi(r) \hat{\phi}.$$

For a coaxial circular path of radius r ,

$$\oint_C \vec{E} \cdot d\vec{l} = 2\pi r E_\phi(r).$$

For $r < a$, the enclosed magnetic flux is

$$\Phi_B(r) = \mu_0 n I_s \pi r^2.$$

Thus,

$$2\pi r E_\phi(r) = -\mu_0 n \pi r^2 \frac{dI_s}{dt},$$

so that

$$\vec{E}(r) = -\frac{\mu_0 n r}{2} \frac{dI_s}{dt} \hat{\phi}, \quad r < a.$$

For $r \geq a$, only the solenoid cross-section contributes to the flux:

$$\Phi_B(r) = \mu_0 n I_s \pi a^2.$$

Consequently,

$$2\pi r E_\phi(r) = -\mu_0 n \pi a^2 \frac{dI_s}{dt},$$

and hence

$$\vec{E}(r) = -\frac{\mu_0 n a^2}{2r} \frac{dI_s}{dt} \hat{\phi}, \quad r \geq a.$$

The two expressions agree at $r = a$, where

$$\vec{E}(a) = -\frac{\mu_0 n a}{2} \frac{dI_s}{dt} \hat{\phi}.$$

At the ring, $r = b$,

$$\vec{E}(b) = -\frac{\mu_0 n a^2}{2b} \frac{dI_s}{dt} \hat{\phi},$$

and its circulation is

$$2\pi b E_\phi(b) = -\mu_0 n \pi a^2 \frac{dI_s}{dt} = \mathcal{E}_r,$$

which consistently reproduces the induced emf.

CONCLUSION:

With positive azimuthal direction chosen counterclockwise as viewed from the $+\hat{z}$ end of the solenoid,

$$I_r = -\frac{\mu_0 n \pi a^2}{R} \frac{dI_s}{dt} \quad (\text{A})$$

and

$$P_J = \frac{\mu_0^2 n^2 \pi^2 a^4}{R} \left(\frac{dI_s}{dt} \right)^2 \quad (\text{W}).$$

The induced electric field is

$$\vec{E}(r) = \begin{cases} -\frac{\mu_0 n r}{2} \frac{dI_s}{dt} \hat{\phi}, & r < a, \\ -\frac{\mu_0 n a^2}{2r} \frac{dI_s}{dt} \hat{\phi}, & r \geq a, \end{cases} \quad (\text{V m}^{-1}).$$

In particular,

$$\vec{E}(b) = -\frac{\mu_0 n a^2}{2b} \frac{dI_s}{dt} \hat{\phi}.$$

For $\frac{dI_s}{dt} < 0$, both $\vec{E}$ and I_r point in $+\hat{\phi}$, and the induced magnetic field opposes the reduction of the original flux. These results apply to an ideal long solenoid and to sufficiently slow current variation that the ring's self-inductance may be neglected.

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Q6. The space between the plates of a parallel-plate capacitor is filled with a dielectric material whose susceptibility varies linearly from 0 at the bottom plate ($x = 0$) to 1 at the top plate ($x = d$). The capacitor is connected to a battery of voltage V . Calculate all the bound charges and check that the total charge is zero. Assume that the battery is connected in such a way that the electric field points along the x -direction while the free charge density, σ_f , is positive at the bottom plate and negative at the top plate.

INTRODUCTION:

The dielectric susceptibility is

$$\chi_e(x) = \frac{x}{d}, \quad 0 \leq x \leq d,$$

so that the position-dependent permittivity is

$$\epsilon(x) = \epsilon_0 \left[1 + \chi_e(x) \right] = \epsilon_0 \left(1 + \frac{x}{d} \right).$$

The dielectric is assumed to be linear, isotropic and locally responding, and fringing fields are neglected. There is no free volume charge within the dielectric. We choose $+\hat{\mathbf{x}}$ from the bottom plate to the top plate. Consistent with the stated orientation,

$$\mathbf{E} = E(x)\hat{\mathbf{x}}, \quad V \equiv \phi(0) - \phi(d) = \int_0^d E(x) dx > 0.$$

The free surface charges on the conducting plates are $+\sigma_f$ at $x = 0$ and $-\sigma_f$ at $x = d$.

The governing relations are

$$\nabla \cdot \mathbf{D} = \rho_f, \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon(x) \mathbf{E},$$

together with

$$\rho_b = -\nabla \cdot \mathbf{P}, \quad \sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}},$$

where $\hat{\mathbf{n}}$ is the outward unit normal from the dielectric.

SOLUTION:

Since $\rho_f = 0$ throughout $0 < x < d$,

$$\frac{dD_x}{dx} = 0.$$

Thus D_x is constant. At the positively charged bottom conductor, the normal component of $\mathbf{D}$ entering the dielectric equals σ_f , and hence

$$\mathbf{D} = \sigma_f \hat{\mathbf{x}}.$$

It follows that the electric field is nonuniform:

$$\mathbf{E}(x) = \frac{\mathbf{D}}{\epsilon(x)} = \frac{\sigma_f}{\epsilon_0(1+x/d)} \hat{\mathbf{x}} = \frac{\sigma_f d}{\epsilon_0(d+x)} \hat{\mathbf{x}}.$$

The applied voltage determines σ_f :

$$V = \int_0^d E(x) dx = \frac{\sigma_f d}{\epsilon_0} \int_0^d \frac{dx}{d+x}.$$

Therefore,

$$V = \frac{\sigma_f d}{\epsilon_0} [\ln(d+x)]_0^d = \frac{\sigma_f d}{\epsilon_0} \ln 2,$$

and

$$\sigma_f = \frac{\epsilon_0 V}{d \ln 2}.$$

The polarization is

$$\mathbf{P}(x) = \epsilon_0 \chi_e(x) \mathbf{E}(x).$$

Substitution gives

$$\mathbf{P}(x) = \epsilon_0 \frac{x}{d} \frac{\sigma_f d}{\epsilon_0(d+x)} \hat{\mathbf{x}} = \frac{\sigma_f x}{d+x} \hat{\mathbf{x}}.$$

The bound volume-charge density is consequently

$$\rho_b(x) = -\nabla \cdot \mathbf{P} = -\frac{dP_x}{dx}.$$

Since

$$\frac{d}{dx} \left(\frac{x}{d+x} \right) = \frac{d}{(d+x)^2},$$

one obtains

$$\rho_b(x) = -\frac{\sigma_f d}{(d+x)^2} = -\frac{\epsilon_0 V}{\ln 2} \frac{1}{(d+x)^2} \quad 0 < x < d.$$

Thus the continuously distributed bound charge is negative everywhere in the dielectric. This is consistent with P_x increasing toward the top plate: a positive polarization gradient produces negative bound volume charge.

At the bottom surface $x = 0$, the outward normal from the dielectric is

$$\hat{\mathbf{n}}_0 = -\hat{\mathbf{x}}.$$

Because $\chi_e(0) = 0$, the polarization there vanishes:

$$\mathbf{P}(0) = 0.$$

Hence

$$\sigma_b(0) = \mathbf{P}(0) \cdot (-\hat{\mathbf{x}}) = 0.$$

At the top surface $x = d$, the outward normal is

$$\hat{\mathbf{n}}_d = +\hat{\mathbf{x}},$$

and

$$\mathbf{P}(d) = \frac{\sigma_f d}{d+d} \hat{\mathbf{x}} = \frac{\sigma_f}{2} \hat{\mathbf{x}}.$$

Therefore,

$$\sigma_b(d) = \mathbf{P}(d) \cdot \hat{\mathbf{x}} = \frac{\sigma_f}{2} = \frac{\epsilon_0 V}{2d \ln 2}.$$

Any lateral surface has its normal perpendicular to $\mathbf{P}$, so its bound surface-charge density is zero within the parallel-plate approximation.

To check the total bound charge, let the plate area be A . The bound volume charge is

$$Q_b^{(\text{vol})} = A \int_0^d \rho_b(x) dx = -A\sigma_f d \int_0^d \frac{dx}{(d+x)^2}.$$

Using

$$\int_0^d \frac{dx}{(d+x)^2} = \left[-\frac{1}{d+x} \right]_0^d = \frac{1}{2d},$$

we find

$$Q_b^{(\text{vol})} = -\frac{A\sigma_f}{2}.$$

The total bound surface charge is

$$Q_b^{(\text{surf})} = A[\sigma_b(0) + \sigma_b(d)] = \frac{A\sigma_f}{2}.$$

Consequently,

$$Q_b^{(\text{total})} = Q_b^{(\text{vol})} + Q_b^{(\text{surf})} = -\frac{A\sigma_f}{2} + \frac{A\sigma_f}{2} = 0.$$

The cancellation is required because polarization merely redistributes the initially neutral dielectric's positive and negative charges.

CONCLUSION:

For the field directed from the positively charged bottom plate toward the negatively charged top plate,

$$\mathbf{E}(x) = \frac{V}{\ln 2} \frac{1}{d+x} \hat{\mathbf{x}}, \quad \mathbf{P}(x) = \frac{\epsilon_0 V}{d \ln 2} \frac{x}{d+x} \hat{\mathbf{x}},$$

and the free surface-charge magnitude is

$$\sigma_f = \frac{\epsilon_0 V}{d \ln 2} \quad \text{in } \text{C m}^{-2}.$$

All bound charges in the one-dimensional parallel-plate approximation are

$$\rho_b(x) = -\frac{\epsilon_0 V}{\ln 2} \frac{1}{(d+x)^2} \quad \text{in } \text{C m}^{-3}, \quad 0 < x < d,$$

$$\sigma_b(0) = 0, \quad \sigma_b(d) = \frac{\epsilon_0 V}{2d \ln 2} \quad \text{in } \text{C m}^{-2},$$

with zero bound charge on the lateral surfaces. The negative volume charge,

$$Q_b^{(\text{vol})} = -\frac{A\epsilon_0 V}{2d \ln 2},$$

is exactly balanced by the positive bound charge on the top dielectric surface,

$$Q_b^{(\text{surf})} = +\frac{A\epsilon_0 V}{2d \ln 2}.$$

Hence the net bound charge is

$$Q_b^{(\text{total})} = 0,$$

as required for an initially neutral dielectric under the stated linear-dielectric and negligible-fringing-field assumptions.

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Q7. Consider a point charge of 5 nC placed at a distance of 1 m from a perfect conducting plane ($z = 0$) of infinite extent. Find the electric field at a point (2, 2, 0) m and show that it is normal to the plane.

INTRODUCTION:

Take the conducting region to be $z \leq 0$ and the physical space containing the charge to be $z > 0$. Since only the perpendicular distance of the charge from the plane is specified, choose the origin at its perpendicular projection on the plane. Thus,

$$q = 5 \times 10^{-9} \text{ C}, \quad \mathbf{r}_q = (0, 0, a), \quad a = 1 \text{ m}.$$

The observation point is

$$P : \quad \mathbf{r}_P = (2, 2, 0) \text{ m}.$$

The electric field requested at the conductor surface means the limiting field immediately outside it, $z = 0^+$; the field inside a perfect conductor is zero. The $+\hat{\mathbf{z}}$ direction is taken from the conductor into the exterior region.

The governing principle is the method of images. The grounded perfect conducting plane is replaced, for $z > 0$, by an image charge

$$q' = -q$$

at the mirror position

$$\mathbf{r}_{q'} = (0, 0, -a).$$

This construction makes the plane $z = 0$ an equipotential of zero potential. By the uniqueness theorem,

the resulting field in $z > 0$ is the physical electrostatic field.

SOLUTION:

For a source charge Q at $\mathbf{r}_0$, Coulomb's law gives

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q(\mathbf{r} - \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|^3}.$$

Let

$$k = \frac{1}{4\pi\epsilon_0} = 8.98755 \times 10^9 \text{ N m}^2 \text{ C}^{-2}.$$

The displacement from the real charge to P is

$$\mathbf{R}_q = \mathbf{r}_P - \mathbf{r}_q = (2, 2, -1) \text{ m},$$

whereas that from the image charge to P is

$$\mathbf{R}_{q'} = \mathbf{r}_P - \mathbf{r}_{q'} = (2, 2, 1) \text{ m}.$$

Their magnitudes are equal:

$$R_q = R_{q'} = \sqrt{2^2 + 2^2 + 1^2} = 3 \text{ m}.$$

Therefore, the field at P , approached from $z > 0$, is

$$\begin{aligned} \mathbf{E}(P) &= k \left[\frac{q\mathbf{R}_q}{R_q^3} + \frac{q'\mathbf{R}_{q'}}{R_{q'}^3} \right] \\ &= \frac{kq}{(3 \text{ m})^3} [(2, 2, -1) - (2, 2, 1)] \text{ m} \\ &= \frac{kq}{27 \text{ m}^3} (0, 0, -2) \text{ m} \\ &= -\frac{2kq}{27 \text{ m}^2} \hat{\mathbf{z}}. \end{aligned}$$

Substituting the numerical values,

$$\begin{aligned} \mathbf{E}(P) &= -\frac{2(8.98755 \times 10^9 \text{ N m}^2 \text{ C}^{-2})(5 \times 10^{-9} \text{ C})}{27 \text{ m}^2} \hat{\mathbf{z}} \\ &= -3.33 \hat{\mathbf{z}} \text{ N C}^{-1}. \end{aligned}$$

The cancellation of tangential components is explicit:

$$E_x = \frac{kq}{27}(2 - 2) = 0, \quad E_y = \frac{kq}{27}(2 - 2) = 0,$$

while

$$E_z = \frac{kq}{27}(-1 - 1) = -\frac{2kq}{27} \neq 0.$$

Since the plane $z = 0$ has normal vector $\hat{\mathbf{z}}$, the field

$$\mathbf{E}(P) = E_z \hat{\mathbf{z}}$$

has no component parallel to the plane and is therefore strictly normal to it.

More generally, at any surface point $(x, y, 0)$,

$$\mathbf{E}(x, y, 0^+) = -\frac{2kqa}{(x^2 + y^2 + a^2)^{3/2}} \hat{\mathbf{z}},$$

which confirms that the tangential cancellation is a consequence of the mirror symmetry and is not peculiar to P .

The dimensions are consistent:

$$\frac{kq}{R^2} = \frac{(\text{N m}^2 \text{ C}^{-2})(\text{C})}{\text{m}^2} = \text{N C}^{-1}.$$

As a physical check, for $x^2 + y^2 \gg a^2$, the surface field decreases as

$$|\mathbf{E}| \sim \frac{2kqa}{(x^2 + y^2)^{3/2}},$$

rather than as the field of an isolated charge; this is consistent with the real charge and its opposite image forming a dipole-like configuration at large distance. The negative sign also has the expected meaning: for $q > 0$, negative charge is induced on the conductor, so the exterior field points toward the conducting plane.

CONCLUSION:

Immediately outside the conducting plane at $(2, 2, 0) \text{ m}$,

$$\mathbf{E}(2, 2, 0^+) = -3.33 \hat{\mathbf{z}} \text{ N C}^{-1}.$$

Thus,

$$E_x = E_y = 0, \quad E_z = -3.33 \text{ N C}^{-1},$$

so the electric field is normal to the plane and directed into the conductor, along $-\hat{\mathbf{z}}$. This result assumes that the charge is at $(0, 0, 1) \text{ m}$, with the origin chosen at its perpendicular projection on the infinite conducting plane.

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Q8. A rectangular coil consists of 50 closely wrapped turns and has dimensions of $0.5 \text{ m} \times 0.4 \text{ m}$. It carries a current of 1.5 A . If a uniform magnetic field $B = 0.1 \text{ T}$ is applied such that the direction of the magnetic field makes an angle of 60° with respect to the plane of the coil, what is the torque exerted on the coil by the magnetic field?

INTRODUCTION:

The demand is to determine both the magnitude and orientation of the magnetic torque on the current-carrying coil. A planar coil of N turns, area A , and current I has magnetic dipole moment

$$\boldsymbol{\mu} = NIA \hat{\mathbf{n}},$$

where $\hat{n}$ is the unit normal to the plane of the coil, with its sense fixed by the right-hand rule relative to the current. In a uniform magnetic field $\mathbf{B}$, the torque is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}, \quad \tau = \mu B \sin \theta,$$

where θ is the angle between the coil normal and $\mathbf{B}$.

The field is stated to make an angle $\alpha = 60^\circ$ with the plane of the coil. Therefore, its angle with the normal is

$$\theta = 90^\circ - \alpha = 30^\circ.$$

The coil is assumed rigid, planar, and sufficiently small that the applied field is uniform over all its turns.

SOLUTION:

The area enclosed by each turn is

$$A = (0.5 \text{ m})(0.4 \text{ m}) = 0.20 \text{ m}^2.$$

Hence, the magnitude of the magnetic dipole moment of the 50-turn coil is

$$\begin{aligned} \mu &= NIA \\ &= (50)(1.5 \text{ A})(0.20 \text{ m}^2) \\ &= 15 \text{ A m}^2. \end{aligned}$$

Using $\theta = 30^\circ$, the torque magnitude is

$$\begin{aligned} \tau &= \mu B \sin \theta \\ &= (15 \text{ A m}^2)(0.1 \text{ T}) \sin 30^\circ \\ &= (15)(0.1) \left(\frac{1}{2} \right) \text{ A m}^2 \text{ T} \\ &= 0.75 \text{ N m}. \end{aligned}$$

Equivalently, since the given angle α is measured from the plane,

$$\tau = NIAB \cos \alpha = (50)(1.5)(0.20)(0.1) \cos 60^\circ = 0.75 \text{ N m}.$$

The dimensional check is

$$[\tau] = [\mu B] = (\text{A m}^2)(\text{T}) = \text{N m},$$

which is the correct dimension of torque.

The torque vector is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}.$$

It is perpendicular to the plane containing $\boldsymbol{\mu}$ and $\mathbf{B}$, with its sense given by the right-hand rule. Since the direction of current around the coil is not specified, the absolute clockwise or anticlockwise sense cannot be assigned. Physically, the torque rotates the coil so as to align $\boldsymbol{\mu}$ with $\mathbf{B}$, thereby making the plane of the coil perpendicular to the magnetic field.

As a limiting check, if the magnetic field were normal to the coil, then $\theta = 0$ and $\tau = 0$, as expected for an already aligned magnetic moment. If the field were parallel to the plane, then $\theta = 90^\circ$ and the torque would attain its maximum value $NIAB$. The calculated value is one-half of this maximum because $\sin 30^\circ = 1/2$.

CONCLUSION:

The torque exerted on the coil is

$$\tau = 0.75 \text{ N m}.$$

Its direction is along $\boldsymbol{\mu} \times \mathbf{B}$, perpendicular to the plane containing the coil normal and the magnetic field. It tends to rotate the coil until its magnetic moment is parallel to $\mathbf{B}$, or equivalently until the plane of the coil is perpendicular to the field. The precise clockwise or anticlockwise sense depends on the unspecified direction of current. This result applies for a rigid planar coil placed in a uniform magnetic field.

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Q9. State and explain Kirchhoff's current law and Kirchhoff's voltage law. Derive these laws from the principles of charge conservation and energy conservation.

INTRODUCTION:

Kirchhoff's laws relate currents and voltages in lumped electrical networks. Kirchhoff's current law follows from conservation of electric charge, while Kirchhoff's voltage law follows from conservation of energy.

We make the lumped-circuit and quasi-static assumptions: circuit elements and junctions have negligible spatial extent compared with the relevant electromagnetic wavelength, connecting wires are ideal unless their resistance is represented explicitly, and no unrepresented time-varying magnetic flux links a circuit loop.

For the current law, consider a small fixed control volume enclosing a junction. A branch current is taken as positive when directed out of this volume. For the voltage law, choose an arbitrary direction of traversal around a closed loop and define

$$\Delta V_k = V_{\text{out},k} - V_{\text{in},k}$$

as the potential change across the k -th element in that direction. Thus, traversal from the negative to the positive terminal of a source gives $+\mathcal{E}$, whereas traversal through a resistor in the direction of current gives $-IR$.

SOLUTION:

1. Kirchhoff's current law

The law states that the algebraic sum of all currents meeting at a junction is zero:

$$\sum_k I_k = 0.$$

With all currents entering the junction taken as positive, the equivalent form is

$$\sum I_{\text{in}} = \sum I_{\text{out}}.$$

The local statement of charge conservation is the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0,$$

where ρ is the volume charge density and $\mathbf{J}$ is the current density.

Integrating over a control volume $\mathcal{V}$ enclosing the junction gives

$$\int_{\mathcal{V}} \frac{\partial \rho}{\partial t} d\tau + \int_{\partial \mathcal{V}} \nabla \cdot \mathbf{J} d\tau = 0.$$

Using the divergence theorem,

$$\frac{d}{dt} \int_{\mathcal{V}} \rho d\tau + \oint_{\partial \mathcal{V}} \mathbf{J} \cdot d\mathbf{a} = 0.$$

If

$$Q_j = \int_{\mathcal{V}} \rho d\tau$$

is the charge stored at the junction, and outward current is positive, then

$$\oint_{\partial \mathcal{V}} \mathbf{J} \cdot d\mathbf{a} = \sum_k I_k.$$

Consequently,

$$\frac{dQ_j}{dt} + \sum_k I_k = 0.$$

In an ordinary lumped circuit, a junction does not continuously accumulate charge. Therefore,

$$\frac{dQ_j}{dt} = 0,$$

and hence

$$\sum_k I_k = 0.$$

If currents entering the junction are assigned positive signs and currents leaving it negative signs, this becomes

$$I_1 + I_2 + \dots - I_3 - I_4 - \dots = 0,$$

or

$$\sum I_{\text{in}} = \sum I_{\text{out}}.$$

The result has a direct physical meaning. If the total incoming current exceeded the outgoing current, positive charge would accumulate at the junction; if the outgoing current exceeded the incoming current, the junction would lose charge. In steady operation neither accumulation can continue, so the two totals must be equal.

The dimensions are consistent:

$$\left[\frac{dQ}{dt} \right] = \text{C s}^{-1} = \text{A} = [I].$$

For a node capable of storing charge, such as a capacitive node, the more general equation

$$\sum_k I_k = -\frac{dQ_j}{dt}$$

must be retained.

2. Kirchhoff's voltage law

The law states that the algebraic sum of all potential changes around any closed circuit loop is zero:

$$\sum_k \Delta V_k = 0.$$

Equivalently, the sum of voltage rises equals the sum of voltage drops:

$$\sum \mathcal{E} = \sum V_{\text{drop}}.$$

To derive the law, let a small positive test charge q make one complete traversal of a closed loop. Because it returns to its initial point, its initial and final electrical states are the same. In steady circuit operation it also has no net change in kinetic energy after a complete circuit. Conservation of energy therefore requires the algebraic sum of the energy changes supplied to and removed from the charge to vanish:

$$\sum_k \Delta U_k = 0.$$

Since voltage is energy transferred per unit charge,

$$\Delta V_k = \frac{\Delta U_k}{q},$$

we obtain

$$q \sum_k \Delta V_k = 0.$$

For $q \neq 0$,

$$\sum_k \Delta V_k = 0.$$

The same result may be written explicitly using node potentials. Suppose a loop passes successively through nodes with potentials

$$V_1, V_2, \dots, V_n, V_1.$$

The total potential change is

$$(V_2 - V_1) + (V_3 - V_2) + \dots + (V_1 - V_n).$$

Every intermediate potential occurs once with a positive sign and once with a negative sign, so the expression telescopes:

$$(V_2 - V_1) + (V_3 - V_2) + \dots + (V_1 - V_n) = 0.$$

Thus,

$$\oint dV = 0.$$

For example, consider a loop containing a source of emf $\mathcal{E}$ and resistors $R_1, \dots, R_n$ carrying the same current I . Traverse the source from its negative to its positive terminal and each resistor in the direction of current. The source produces the voltage rise

$$+\mathcal{E},$$

while Ohm's law gives the voltage change across each resistor as

$$-IR_k.$$

Kirchhoff's voltage law then gives

$$\mathcal{E} - IR_1 - IR_2 - \dots - IR_n = 0,$$

so that

$$\mathcal{E} = I \sum_{k=1}^n R_k.$$

Multiplying by the charge q shows the energy balance directly:

$$q\mathcal{E} = \sum_{k=1}^n qIR_k.$$

Multiplying instead by I gives the corresponding power balance:

$$I\mathcal{E} = \sum_{k=1}^n I^2 R_k.$$

Thus the power supplied by the source equals the power dissipated in the resistors, as required by conservation of energy.

The dimensional check is

$$[\mathcal{E}] = [IR] = A \Omega = V = J C^{-1}.$$

The sign convention also gives the correct limiting case: if all resistances tend to zero while a finite ideal emf remains, the equation would require an unbounded current. This signals that a real loop must contain resistance, internal source resistance, inductance, or another element that limits the current.

If a time-dependent magnetic flux Φ_B links the loop, the electric field is not conservative. Faraday's law gives

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}.$$

Therefore the simple form $\sum_k \Delta V_k = 0$ applies only when the linked magnetic flux is constant or when the induced emf

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}$$

is included explicitly as a circuit emf. With every source, including induction, represented consistently, the generalized energy balance is again

$$\sum \mathcal{E} - \sum V_{\text{drop}} = 0.$$

CONCLUSION:

Kirchhoff's current law is

$$\sum_k I_k = 0 \quad \text{or} \quad \sum I_{\text{in}} = \sum I_{\text{out}},$$

where current is measured in amperes and its sign is fixed by the declared direction at the junction. It is the lumped-circuit consequence of charge conservation and is valid when the junction has no net charge accumulation. More generally,

$$\frac{dQ_j}{dt} + \sum_k I_k = 0.$$

Kirchhoff's voltage law is

$$\sum_k \Delta V_k = 0 \quad \text{or} \quad \sum \mathcal{E} = \sum V_{\text{drop}},$$

where voltage is measured in volts and each sign is determined by the chosen direction of loop traversal. It expresses conservation of energy: a charge returning to its starting point has zero net energy change, so energy supplied per unit charge equals energy absorbed or dissipated per unit charge.

The ordinary voltage law requires a lumped, quasi-static circuit with no omitted time-varying magnetic flux. If such a flux is present, its induced emf

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}$$

must be included. Hence Kirchhoff's laws express, respectively, the impossibility of sustained charge creation or loss at a junction and the impossibility of net energy gain or loss in one complete circuit traversal.

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Q10. A parallel plate capacitor having circular plates of radius 10 cm is being charged. If the electric field at any instant within the capacitor changes at the rate $5.0 \text{ V m}^{-1} \text{ s}^{-1}$, calculate the magnetic intensity $|\vec{H}|$ inside the capacitor.

INTRODUCTION:

The changing electric field between the plates produces a displacement current and hence a magnetic field. The appropriate governing principle is the Ampere–Maxwell law,

$$\oint_C \vec{H} \cdot d\vec{l} = \frac{d}{dt} \int_S \vec{D} \cdot d\vec{S}, \quad \vec{D} = \epsilon_0 \vec{E}$$

for air or vacuum between the plates.

Let the circular plates have radius

$$a = 10 \text{ cm} = 0.10 \text{ m},$$

and let $r \leq a$ denote the perpendicular distance from their common axis. Fringing is neglected, so the electric field is taken as uniform and perpendicular to the plates. By cylindrical symmetry, $\vec{H}$ is tangential to a circle of radius r and has constant magnitude on that circle. Its direction is determined by the right-hand rule with respect to the displacement-current direction. The stated rate is interpreted as

$$\left| \frac{\partial E}{\partial t} \right| = 5.0 \text{ V m}^{-1} \text{ s}^{-1}.$$

SOLUTION:

Choose a circular Amperian loop of radius $r \leq a$, concentric with the plates. Since $\vec{H}$ is tangential and uniform along this loop,

$$\oint_C \vec{H} \cdot d\vec{l} = H(r)(2\pi r).$$

The electric flux through the surface bounded by the loop is

$$\Phi_E = E(\pi r^2).$$

Therefore, the displacement current enclosed by the loop is

$$I_d(r) = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \pi r^2 \frac{\partial E}{\partial t}.$$

Applying the Ampere–Maxwell law,

$$H(r)(2\pi r) = \epsilon_0 \pi r^2 \left| \frac{\partial E}{\partial t} \right|.$$

Thus, at any radial position inside the plate area,

$$H(r) = \frac{\epsilon_0 r}{2} \left| \frac{\partial E}{\partial t} \right|, \quad 0 \leq r \leq a.$$

Substituting the given rate,

$$H(r) = \frac{(8.854 \times 10^{-12} \text{ F m}^{-1})r}{2} (5.0 \text{ V m}^{-1} \text{ s}^{-1}),$$

or

$$H(r) = 2.21 \times 10^{-11} r \text{ A m}^{-1}$$

when r is expressed in metres.

Since the question specifies the plate radius but no separate observation radius, the intended value is the maximum field inside the capacitor, attained near the plate rim at $r = a = 0.10 \text{ m}$:

$$\begin{aligned} H(a) &= \frac{(8.854 \times 10^{-12})(0.10)(5.0)}{2} \text{ A m}^{-1} \\ &= 2.2135 \times 10^{-12} \text{ A m}^{-1} \\ &\approx 2.2 \times 10^{-12} \text{ A m}^{-1}. \end{aligned}$$

The dimensional check is

$$\left[\epsilon_0 r \frac{\partial E}{\partial t} \right] = \frac{\text{C}}{\text{V m}} \text{ m} \frac{\text{V}}{\text{m s}} = \frac{\text{C}}{\text{m s}} = \frac{\text{A}}{\text{m}},$$

which is the correct dimension of magnetic intensity. Also,

$$\lim_{r \rightarrow 0} H(r) = 0,$$

as required by cylindrical symmetry, and $H(r)$ increases linearly from the axis to the rim.

CONCLUSION:

The magnetic intensity at a distance r from the common axis, within the circular plate area, is

$$|\vec{H}(r)| = \frac{\epsilon_0 r}{2} \left| \frac{\partial E}{\partial t} \right| = 2.21 \times 10^{-11} r \text{ A m}^{-1}, \quad 0 \leq r \leq 0.10 \text{ m}.$$

At the rim, where the magnitude is maximum,

$$|\vec{H}|_{\text{max}} = 2.2 \times 10^{-12} \text{ A m}^{-1}.$$

The field circulates azimuthally around the capacitor axis, with its sense given by the right-hand rule relative to the direction of the increasing electric field. This result assumes a uniform electric field and neglects fringing near the plate edge.



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