



UPSC PHYSICS OPTIONAL for CSE & IFoS

ABHI PHYSICS

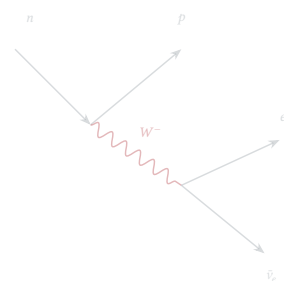
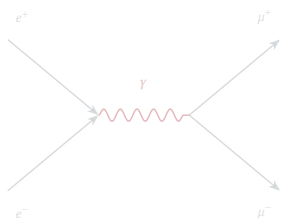
$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$$

$$[\hat{x}, \hat{p}] = i\hbar$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

PREVIOUS YEAR QUESTIONS



41 YEARS QUESTION BANK | 1986-2026

CSE + IFoS

SUBJECT

Atomic and Molecular Physics

10 SOLVED QUESTIONS • CSE + IFoS

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$E^2 = p^2 c^2 + m^2 c^4$$

$$\partial_\mu j^\mu = 0$$

SUBJECT-WISE • TOPIC-WISE • SOURCE-TAGGED

For Civil Services and Indian Forest Service Examinations

Sample Preview extbar 10 Solved Questions

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CSE • 2026 • 10 MARKS

Q1. In a Stern–Gerlach experiment, the magnetic field varies with distance in the z -direction according to $\frac{dB_z}{dz} = 1.5 \text{ T/mm}$. The speed of silver atoms coming from the oven is 725 m/s . Silver atoms travel a distance of 3 cm through the magnet. Calculate the separation of the two beams of atoms when they leave the magnet. (4pt) [Given : The mass of a silver atom $= 1.8 \times 10^{-25} \text{ kg}$ and its magnetic moment is 1 Bohr magneton.]

INTRODUCTION:

The required quantity is the transverse separation of the two silver-atom beams at the exit of the magnet. Let the incident beam travel along the x -direction and let $+z$ be the direction in which B_z increases, so that

$$\frac{dB_z}{dz} > 0.$$

In an ideal Stern–Gerlach magnet, an atom with magnetic-moment component μ_z has magnetic potential energy

$$U = -\mu_z B_z$$

and therefore experiences the force

$$F_z = -\frac{\partial U}{\partial z} = \mu_z \frac{dB_z}{dz}.$$

For silver atoms the two allowed moment components are taken as

$$\mu_z = \pm \mu_B, \quad \mu_B = 9.27 \times 10^{-24} \text{ J T}^{-1}.$$

We assume a constant field gradient throughout the 3 cm magnet, negligible initial velocity in the z -direction, unchanged longitudinal speed, and negligible fringe-field and gravitational effects.

SOLUTION:

First convert the magnetic-field gradient to SI units:

$$\frac{dB_z}{dz} = 1.5 \text{ T mm}^{-1} = 1.5 \times 10^3 \text{ T m}^{-1}.$$

Thus the two magnetic-moment orientations experience equal and opposite forces,

$$F_{z,\pm} = \pm \mu_B \frac{dB_z}{dz},$$

whose common magnitude is

$$F = (9.27 \times 10^{-24} \text{ J T}^{-1})(1.5 \times 10^3 \text{ T m}^{-1}) = 1.39 \times 10^{-20} \text{ N}.$$

The corresponding transverse accelerations are

$$a_{z,\pm} = \frac{F_{z,\pm}}{m} = \pm \frac{\mu_B}{m} \frac{dB_z}{dz},$$

so their common magnitude is

$$a = \frac{1.39 \times 10^{-20} \text{ N}}{1.8 \times 10^{-25} \text{ kg}} = 7.73 \times 10^4 \text{ m s}^{-2}.$$

The time spent inside the magnet is determined by the longitudinal motion:

$$t = \frac{L}{v} = \frac{3.0 \times 10^{-2} \text{ m}}{725 \text{ m s}^{-1}} = 4.14 \times 10^{-5} \text{ s}.$$

Since the initial transverse velocity is zero, the displacement of each component is

$$z_{\pm} = \frac{1}{2} a_{z,\pm} t^2.$$

Hence the two displacements are equal in magnitude and opposite in direction. Their separation at the magnet exit is therefore

$$\Delta z = z_+ - z_- = at^2 = \frac{\mu_B}{m} \frac{dB_z}{dz} \left(\frac{L}{v} \right)^2.$$

Substituting the data,

$$\begin{aligned} \Delta z &= \frac{(9.27 \times 10^{-24} \text{ J T}^{-1})(1.5 \times 10^3 \text{ T m}^{-1})}{1.8 \times 10^{-25} \text{ kg}} \left(\frac{3.0 \times 10^{-2} \text{ m}}{725 \text{ m s}^{-1}} \right)^2 \\ &= (7.73 \times 10^4 \text{ m s}^{-2})(4.14 \times 10^{-5} \text{ s})^2 \\ &= 1.32 \times 10^{-4} \text{ m}. \end{aligned}$$

Equivalently, each beam is displaced from the undeflected axis by

$$|z_{\pm}| = \frac{\Delta z}{2} = 6.62 \times 10^{-5} \text{ m}.$$

The component with $\mu_z = +\mu_B$ moves toward $+z$, the direction of increasing field, whereas the component with $\mu_z = -\mu_B$ moves toward $-z$. The result also has the expected dependence $\Delta z \propto (dB_z/dz)L^2/v^2$: the splitting vanishes for zero field gradient and increases for a longer interaction time.

CONCLUSION:

The separation of the two silver-atom beams as they leave the magnet is

$$\Delta z = 1.3 \times 10^{-4} \text{ m} \approx 0.13 \text{ mm}.$$

The two beams are separated along the z -direction: the $\mu_z = +\mu_B$ component is deflected toward $+z$, while the $\mu_z = -\mu_B$ component is deflected toward $-z$. Each beam is approximately $6.6 \times 10^{-5} \text{ m}$ from the original beam axis. This result applies for a uniform field gradient, negligible initial transverse velocity and fringe fields, and approximately constant longitudinal speed; the opposite deflections demonstrate the two allowed orientations of the silver atom's magnetic moment.

CSE • 2026 • 8 MARKS

Q2. Why can the experimental observation of the Lamb shift not be explained by the Dirac theory?

INTRODUCTION:

The Lamb shift is the experimentally observed separation of atomic energy levels that the one-electron Dirac theory predicts to be degenerate, most notably the $2S_{1/2}$ and $2P_{1/2}$ levels of hydrogen. The question therefore requires a comparison between the Dirac spectrum in a static Coulomb field and the experimentally observed spectrum.

We assume an infinitely heavy, pointlike nucleus producing the Coulomb potential

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}.$$

In the Dirac theory the electron is relativistic, but the electromagnetic field is treated as a prescribed classical field. Consequently, electron spin and relativistic effects are included, whereas the quantized electromagnetic vacuum, radiative self-energy, vacuum polarization, and nuclear recoil or finite-size corrections are absent.

SOLUTION:

The stationary Dirac equation for an electron in the nuclear Coulomb field is

$$[c \boldsymbol{\alpha} \cdot \mathbf{p} + \beta mc^2 + V(r)] \psi = E\psi.$$

Its bound-state energy eigenvalues may be written as

$$E_{nj} = mc^2 \left[1 + \frac{(Z\alpha)^2}{\left(n - j - \frac{1}{2} + \sqrt{\left(j + \frac{1}{2} \right)^2 - (Z\alpha)^2} \right)^2} \right]^{-1/2},$$

where n is the principal quantum number, j is the total electronic angular-momentum quantum number, and

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

is the fine-structure constant.

A decisive feature of this result is that the Coulomb-Dirac energy depends on n and j , but not separately on the orbital angular-momentum quantum number l . For the two states

$$2S_{1/2} : \quad n = 2, \quad l = 0, \quad j = \frac{1}{2},$$

and

$$2P_{1/2} : \quad n = 2, \quad l = 1, \quad j = \frac{1}{2},$$

the values of n and j are identical. Hence the Dirac formula gives

$$E_{2S_{1/2}}^{\text{Dirac}} = E_{2P_{1/2}}^{\text{Dirac}},$$

or

$$\Delta E_{\text{Dirac}} = E_{2S_{1/2}}^{\text{Dirac}} - E_{2P_{1/2}}^{\text{Dirac}} = 0.$$

This degeneracy is not merely a consequence of a low-order approximation. It is an exact property of the Dirac equation for an electron in an ideal $1/r$ Coulomb potential. The Dirac theory successfully accounts for the leading relativistic fine structure, including spin-orbit coupling and the relativistic correction to the kinetic energy, but these contributions still leave states with the same n and j degenerate.

Experimentally, however, the $2S_{1/2}$ level lies above the $2P_{1/2}$ level by a small but nonzero amount:

$$E_{2S_{1/2}} - E_{2P_{1/2}} > 0.$$

This observed separation is the Lamb shift. Since the uncorrected Dirac theory predicts the separation to be exactly zero, the observation cannot be explained within that theory.

The missing physics appears when the electromagnetic field is quantized. In quantum electrodynamics, an atomic electron interacts continually with vacuum electromagnetic fluctuations. The principal radiative corrections are:

1. The electron self-energy: the electron emits and reabsorbs virtual photons, thereby changing its bound-state energy.
2. Vacuum polarization: virtual electron-positron pairs modify the effective Coulomb interaction between the nucleus and the electron.

These corrections depend on the spatial form of the electron wavefunction and therefore distinguish states having the same n and j but different l . In particular, an S -state wavefunction is nonzero at the nucleus,

$$|\psi_{nS}(0)|^2 \neq 0,$$

whereas a P -state wavefunction vanishes there,

$$|\psi_{nP}(0)|^2 = 0.$$

Short-distance radiative modifications of the electron-nucleus interaction consequently affect the $2S_{1/2}$ state more strongly than the $2P_{1/2}$ state. The degeneracy of the Coulomb-Dirac spectrum is therefore removed.

Schematically, the physical energies have the form

$$E_{nlj} = E_{nj}^{\text{Dirac}} + \Delta E_{nlj}^{\text{self-energy}} + \Delta E_{nlj}^{\text{vacuum polarization}} + \dots,$$

so that

$$E_{2S_{1/2}} - E_{2P_{1/2}} = \Delta E_{2S_{1/2}}^{\text{QED}} - \Delta E_{2P_{1/2}}^{\text{QED}} \neq 0.$$

The experimentally observed sign, with the $2S_{1/2}$ level shifted upward relative to $2P_{1/2}$, is reproduced by the full radiative calculation.

CONCLUSION:

The Dirac equation in a static Coulomb potential predicts that energy depends only on n and j . It therefore gives

$$E_{2S_{1/2}}^{\text{Dirac}} = E_{2P_{1/2}}^{\text{Dirac}}, \quad \Delta E_{\text{Dirac}} = 0.$$

Experiment instead finds a nonzero splitting, with $2S_{1/2}$ above $2P_{1/2}$. Thus the Lamb shift lies beyond the one-particle Dirac theory and demonstrates the necessity of quantum-electrodynamic radiative corrections, chiefly electron self-energy and vacuum polarization.

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Q3. Calculate the number of permitted electrons in a sub-shell and a shell in an atom.

INTRODUCTION:

The maximum number of electrons permitted in an atomic state is determined by the quantum numbers and the Pauli exclusion principle: no two electrons in an atom can have the same set of four quantum numbers

$$(n, l, m_l, m_s).$$

Here $n = 1, 2, 3, \dots$ is the principal quantum number, $l = 0, 1, \dots, n-1$ is the orbital angular-momentum quantum number, $m_l = -l, -l+1, \dots, l$ is the magnetic quantum number, and $m_s = \pm \frac{1}{2}$ is the spin quantum number. A sub-shell is specified by fixed n and l , whereas a shell is specified only by fixed n .

SOLUTION:

For a sub-shell of orbital quantum number l , the allowed values of m_l are

$$-l, -l+1, \dots, 0, \dots, l-1, l.$$

Hence the number of distinct spatial orbitals in that sub-shell is

$$N_{\text{orb}} = l - (-l) + 1 = 2l + 1.$$

For each orbital, m_s can take the two values $+\frac{1}{2}$ and $-\frac{1}{2}$. Thus, by the Pauli exclusion principle, each orbital can accommodate at most two electrons. Therefore,

$$N_{\text{sub-shell}} = 2(2l + 1).$$

Consequently, the maximum occupancies of the s, p, d , and f sub-shells are respectively

Sub-shell	l	$2(2l + 1)$
s	0	2
p	1	6
d	2	10
f	3	14

For a shell with principal quantum number n , the allowed sub-shells have

$$l = 0, 1, 2, \dots, n-1.$$

The maximum number of electrons in the shell is therefore the sum of the capacities of all its sub-shells:

$$N_{\text{shell}} = \sum_{l=0}^{n-1} 2(2l + 1).$$

Using

$$\sum_{l=0}^{n-1} (2l + 1) = 2 \frac{n(n-1)}{2} + n = n^2,$$

we obtain

$$N_{\text{shell}} = 2n^2.$$

For example, the $n = 1, 2, 3$, and 4 shells can contain at most

$$2, \quad 8, \quad 18, \quad 32$$

electrons, respectively. For $n = 1$, only $l = 0$ is permitted, so the shell capacity $2n^2 = 2$ correctly agrees with the $1s$ sub-shell capacity.

CONCLUSION:

A sub-shell with orbital quantum number l contains $2l + 1$ orbitals and can accommodate a maximum of

$$2(2l + 1) \text{ electrons}.$$

A shell with principal quantum number n , containing all sub-shells $l = 0$ to $n-1$, can accommodate a maximum of

$$2n^2 \text{ electrons}.$$

These maximum occupancies follow directly from the allowed quantum-number states and the Pauli exclusion principle.

CSE • 2026 • 12 MARKS

Q4. Explain how Pauli's exclusion principle helps in determining the electronic configuration in a many-electron system.

INTRODUCTION:

An electronic configuration specifies how the electrons of an atom are distributed among its orbitals. In a many-electron atom, a one-electron state is labelled by four quantum numbers:

$$(n, l, m_l, m_s),$$

where n is the principal quantum number, l the orbital angular-momentum quantum number, m_l its magnetic quantum number, and $m_s = \pm \frac{1}{2}$ the spin projection.

Electrons are identical spin- $\frac{1}{2}$ fermions, so their total many-electron wavefunction must be antisymmetric under the interchange of any two electrons. This requirement gives Pauli's exclusion principle:

No two electrons in an atom can have the same set of all four quantum numbers.

The principle determines the maximum allowed occupation of every orbital and subshell. The actual order of filling is additionally governed by orbital energies, while the distribution among degenerate orbitals is governed by Hund's rules.

SOLUTION:

For a specified spatial orbital (n, l, m_l) , the only remaining quantum number is the spin projection. Since an electron can have only

$$m_s = +\frac{1}{2} \quad \text{or} \quad m_s = -\frac{1}{2},$$

at most two electrons can occupy that spatial orbital. If two electrons occupy it, their spins must be opposite:

$$(n, l, m_l, +\frac{1}{2}), \quad (n, l, m_l, -\frac{1}{2}).$$

A third electron would necessarily repeat one of these two complete sets of quantum numbers and is therefore forbidden.

For a subshell with orbital quantum number l ,

$$m_l = -l, -l+1, \dots, l,$$

so there are $2l+1$ distinct spatial orbitals. Each orbital can contain two electrons; hence the maximum capacity of the subshell is

$$N_l = 2(2l+1).$$

Thus,

subshell	l	maximum number of electrons
s	0	2
p	1	6
d	2	10
f	3	14

For a shell of principal quantum number n , the allowed values are

$$l = 0, 1, \dots, n-1.$$

Therefore its maximum electronic capacity is

$$N_n = \sum_{l=0}^{n-1} 2(2l+1) = 2 \sum_{l=0}^{n-1} (2l+1) = 2n^2,$$

because the sum of the first n odd integers is n^2 . Hence the K , L , and M shells can accommodate at most 2, 8, and 18 electrons, respectively.

To construct the ground-state configuration of an atom with atomic number Z , its Z electrons are placed successively in the available one-electron states:

1. Orbitals are filled approximately in increasing order of energy, according to the Aufbau principle.
2. Pauli's principle restricts each orbital to at most two electrons with opposite spins.
3. When several orbitals of the same subshell are degenerate, Hund's first rule places electrons singly with parallel spins before pairing occurs, thereby reducing electron-electron repulsion and lowering the energy.

For example, carbon has $Z = 6$. Filling the lowest-energy orbitals gives

$$1s^2 2s^2 2p^2.$$

The $1s$ and $2s$ orbitals each contain two electrons of opposite spin, as required by Pauli's principle. The two $2p$ electrons occupy two different p -orbitals with parallel spins in the ground state:

$$2p_x^1 2p_y^1 2p_z^0,$$

up to an equivalent choice of the degenerate p -orbitals. Pauli's principle forbids placing more than two electrons in any one of these orbitals; Hund's rule determines the energetically preferred distribution before pairing.

The wavefunction basis of this restriction can be seen from a two-electron antisymmetric state,

$$\Psi(1, 2) = \frac{1}{\sqrt{2}} [\phi_a(1)\phi_b(2) - \phi_a(2)\phi_b(1)].$$

If the two electrons were assigned the same spin-orbital, $\phi_a = \phi_b = \phi$, then

$$\Psi(1, 2) = \frac{1}{\sqrt{2}} [\phi(1)\phi(2) - \phi(2)\phi(1)] = 0.$$

Thus a physical many-electron state cannot contain two electrons in the same spin-orbital. This is why Pauli's principle fixes the allowed occupancies and gives the characteristic shell structure underlying the periodic table.

CONCLUSION:

Pauli's exclusion principle determines electronic configurations by allowing each spin-orbital to be occupied by at most one electron, or equivalently each spatial orbital by at most two electrons with opposite spins. Consequently, a subshell with quantum number l holds at most $2(2l + 1)$ electrons and a shell with principal quantum number n holds at most $2n^2$ electrons. Together with the orbital-energy ordering and Hund's rules, this restriction uniquely specifies the allowed ground-state distribution of electrons and accounts for atomic shell structure, valency, and the periodic recurrence of chemical properties.

CSE • 2025 • 15 MARKS

Q5. The Stern-Gerlach experiment is a landmark experiment in quantum mechanics. Discuss about the most important findings of this experiment.

INTRODUCTION:

The Stern-Gerlach experiment tests whether the component of an atomic magnetic moment along a chosen direction can vary continuously, as expected classically, or can assume only discrete quantum values. A collimated beam of neutral atoms is passed through a strongly inhomogeneous magnetic field and the spatial distribution of the emerging atoms is recorded.

Choose a laboratory frame in which the atomic beam travels along $+x$. Near the beam axis, let the magnetic field be predominantly along z ,

$$\mathbf{B} \simeq B_z(z) \hat{\mathbf{z}}, \quad G \equiv \frac{\partial B_z}{\partial z} > 0.$$

Thus increasing z is the direction of increasing field magnitude. This convention will be used for all force and path assignments.

The atoms are assumed to move independently in vacuum. Their centre-of-mass motion may be treated semiclassically, whereas their internal angular momentum is treated quantum mechanically. The field direction is assumed to vary sufficiently slowly along an atomic trajectory that the magnetic-moment eigenstate follows the local field direction. Nuclear magnetic moments and other small corrections are neglected in comparison with the electronic magnetic moment. Neutral atoms are used so that there is no ordinary Lorentz force $\mathbf{q}\mathbf{v} \times \mathbf{B}$.

The governing interaction between an atomic magnetic moment $\boldsymbol{\mu}$ and the field is

$$H_{\text{int}} = -\boldsymbol{\mu} \cdot \mathbf{B}.$$

The experiment therefore converts the allowed eigenvalues of the magnetic-moment component into distinguishable spatial deflections.

SOLUTION:

For a moment eigenstate having a definite component μ_z along the local field, its magnetic interaction energy is

$$E(z) = -\mu_z B_z(z).$$

The corresponding force follows from the spatial derivative of the energy:

$$F_z = -\frac{\partial E}{\partial z} = \mu_z \frac{\partial B_z}{\partial z} = \mu_z G.$$

Consequently, a uniform magnetic field may produce a torque or a Zeeman energy splitting, but it cannot separate the beam spatially because $G = 0$. The inhomogeneity of the field is essential.

Classically, the magnetic moments in an unpolarized beam could be oriented at any angle θ to the z -axis. Their components would then form a continuous range,

$$\mu_z = \mu \cos \theta, \quad -\mu \leq \mu_z \leq \mu,$$

and hence

$$-\mu G \leq F_z \leq \mu G.$$

The detector should therefore show a continuously broadened band rather than sharply separated components.

Quantum mechanically, an atom with total electronic angular momentum $\mathbf{J}$ has

$$J^2 = J(J+1)\hbar^2, \quad J_z = m_J \hbar,$$

where

$$m_J = -J, -J+1, \dots, J.$$

For an electronic magnetic moment,

$$\boldsymbol{\mu} = -g_J \mu_B \frac{\mathbf{J}}{\hbar}, \quad \mu_B = \frac{e\hbar}{2m_e},$$

where g_J is the Lande factor and μ_B is the Bohr magneton. Hence

$$\mu_z = -g_J \mu_B m_J$$

and the allowed forces are

$$F_z = -g_J \mu_B m_J G.$$

There are only $2J+1$ possible values of m_J , so an ideal beam separates into $2J+1$ discrete components.

Stern and Gerlach used silver atoms. The ground-state electronic configuration is

$$[\text{Kr}] 4d^{10} 5s^1.$$

The closed shells make no net contribution to the electronic angular momentum. For the single 5s electron,

$$L = 0, \quad S = \frac{1}{2}, \quad J = \frac{1}{2},$$

so the silver ground state is a two-state system with

$$m_J = +\frac{1}{2} \quad \text{or} \quad m_J = -\frac{1}{2}.$$

Since $L = 0$, the Lande factor is approximately

$$g_J \approx 2.$$

Therefore,

$$\begin{aligned} m_J = +\frac{1}{2} : \quad & \mu_z = -\mu_B, \quad F_z = -\mu_B G, \\ m_J = -\frac{1}{2} : \quad & \mu_z = +\mu_B, \quad F_z = +\mu_B G. \end{aligned}$$

With the convention $G > 0$, the $m_J = +\frac{1}{2}$ state is deflected toward decreasing z , while the $m_J = -\frac{1}{2}$ state is deflected toward increasing z . This reversal between angular momentum and magnetic-moment directions occurs because the electron has negative charge. In the limit $G \rightarrow 0$, both forces vanish, as required for a uniform field.

The most important findings and implications are as follows.

1. The observed silver beam divided into two distinct components instead of forming the continuous band predicted by classical orientation theory. This supplied direct evidence for *space quantization*: only discrete projections of atomic angular momentum and magnetic moment are obtained along the measurement axis.

The quantization axis is not a permanently preferred direction in space. It is selected by the orientation of the Stern–Gerlach magnetic field.

2. The twofold splitting is explained in modern quantum mechanics by the intrinsic spin of the unpaired 5s electron:

$$S = \frac{1}{2}, \quad S_z = \pm \frac{\hbar}{2}.$$

The experiment was performed before electron spin had been introduced, so it was originally interpreted as evidence for quantized atomic angular momentum. Its modern interpretation establishes the characteristic two-valuedness of a spin- $\frac{1}{2}$ system.

3. The result shows that intrinsic spin is not an ordinary classical rotation of an extended body. For any chosen measurement axis, a spin- $\frac{1}{2}$ particle yields only two component values, even though a classical angular-momentum vector could have a continuous range of projections.

4. A Stern–Gerlach apparatus acts as a spin analyzer. If a general incident spin state is written as

$$|\psi\rangle = a|+z\rangle + b|-z\rangle, \quad |a|^2 + |b|^2 = 1,$$

where

$$S_z|\pm z\rangle = \pm \frac{\hbar}{2}|\pm z\rangle,$$

then the two separated paths occur with probabilities

$$P(+z) = |a|^2, \quad P(-z) = |b|^2.$$

Here the labels $+z$ and $-z$ denote spin eigenvalues, not spatial deflection directions. For $G > 0$,

$$\begin{aligned} | +z \rangle &\longrightarrow \text{path toward decreasing } z \\ | -z \rangle &\longrightarrow \text{path toward increasing } z. \end{aligned}$$

If one output path is selected and the other is blocked, the apparatus also prepares the atoms in a definite spin eigenstate.

5. Successive Stern–Gerlach measurements demonstrate the incompatibility of spin components along different axes. An atom prepared in $|+z\rangle$ gives the same definite result when measured again along z . However,

$$|+z\rangle = \frac{1}{\sqrt{2}}(|+x\rangle + |-x\rangle),$$

so a subsequent measurement along x produces the two S_x values with equal probabilities:

$$P(+x) = P(-x) = \frac{1}{2}.$$

This follows from the angular-momentum commutation relation

$$[S_x, S_z] = -i\hbar S_y \neq 0.$$

Thus S_x and S_z cannot simultaneously possess sharp values.

6. The experiment demonstrates how a quantum internal degree of freedom becomes correlated with a macroscopic spatial path. The inhomogeneous field produces the association

$$\begin{aligned} m_J = +\frac{1}{2} &\longleftrightarrow F_z = -\mu_B G \\ m_J = -\frac{1}{2} &\longleftrightarrow F_z = +\mu_B G. \end{aligned}$$

This state-dependent separation is the basic principle behind spin filtering, atomic-beam magnetic-resonance techniques and many quantum-state measurements.

Sharp resolution of the two components requires the field gradient and interaction time to be sufficiently large, while the incident beam must be sufficiently well collimated that the separation exceeds the spread caused by the initial transverse velocities. The atomic magnetic state must also remain correlated with the local field direction during passage through the magnet. These conditions distinguish the quantum splitting from ordinary geometrical broadening of the beam.

CONCLUSION:

The decisive observation of the Stern–Gerlach experiment is that a neutral silver-atom beam separates into two discrete components rather than a continuous distribution. For the silver ground state,

$$\begin{aligned} J &= S = \frac{1}{2} \\ m_J &= \pm \frac{1}{2} \\ \mu_z &= -g_J \mu_B m_J \\ g_J &\approx 2 \end{aligned}$$

where μ_B is measured in J T^{-1} . With

$$G = \frac{\partial B_z}{\partial z} > 0,$$

the two force eigenvalues, measured in newtons, are

$$m_J = +\frac{1}{2} : F_z = -\mu_B G \quad \text{toward decreasing } z,$$

and

$$m_J = -\frac{1}{2} : F_z = +\mu_B G \quad \text{toward increasing } z.$$

Provided that the beam is adequately collimated, the magnetic-field gradient is sufficiently strong, and the internal state remains correlated with the local field, the experiment establishes discrete angular-momentum projections and the two-valued spin- $\frac{1}{2}$ degree of freedom. Its broader inference is that quantum measurement is probabilistic and axis-dependent: a Stern–Gerlach apparatus can analyse and prepare spin states, while spin components along different axes are incompatible because their operators do not commute. Atomic angular momentum is therefore intrinsically quantum mechanical and cannot be represented by a classical continuous distribution of magnetic-moment orientations.

CSE • 2025 • 10 MARKS

Q6. Draw the normal Zeeman pattern for $^1F_3 - ^1D_2$ transition.

INTRODUCTION:

The magnetic field is taken along the $+z$ -direction,

$$\mathbf{B} = B\hat{z}, \quad B > 0,$$

and the weak-field Zeeman regime is assumed, so that the spin-orbit coupling remains intact and L, S, J, M are good quantum numbers. The first-order magnetic interaction is

$$H_Z = \frac{\mu_B}{\hbar} (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B},$$

giving the level shift

$$\Delta E = \mu_B g_J B M,$$

where

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

Let M_u and M_l denote the magnetic quantum numbers of the upper and lower levels. The sign convention used for a component is

$$q = M_u - M_l.$$

For an electric-dipole transition,

$$q = 0, \pm 1.$$

The $q = 0$ component is denoted by π , while $q = +1$ and $q = -1$ are denoted by σ_+ and σ_- , respectively.

The required pattern is obtained by finding the magnetic sublevels, the allowed transitions between them, and the corresponding frequency and polarization of each distinct component.

SOLUTION:

For both terms the multiplicity is $2S + 1 = 1$, hence

$$S = 0.$$

Therefore $J = L$, and the Lande factor reduces to

$$g_J = 1 + \frac{J(J+1) - L(L+1)}{2J(J+1)} = 1.$$

Thus

$$g_u = g(^1F_3) = 1, \quad g_l = g(^1D_2) = 1,$$

which is the defining condition for a normal Zeeman pattern.

The upper level has

$$J_u = 3, \quad M_u = -3, -2, -1, 0, 1, 2, 3,$$

and therefore splits into seven equally spaced sublevels:

$$E_u(M_u) = E_u^{(0)} + \mu_B B M_u.$$

The lower level has

$$J_l = 2, \quad M_l = -2, -1, 0, 1, 2,$$

and splits into five equally spaced sublevels:

$$E_l(M_l) = E_l^{(0)} + \mu_B B M_l.$$

If

$$\nu_0 = \frac{E_u^{(0)} - E_l^{(0)}}{h}$$

is the zero-field transition frequency, an allowed Zeeman transition has frequency

$$\begin{aligned} \nu(M_u, M_l) &= \frac{E_u(M_u) - E_l(M_l)}{h} \\ &= \nu_0 + \frac{\mu_B B}{h} (M_u - M_l). \end{aligned}$$

Defining

$$\nu_L = \frac{\mu_B B}{h},$$

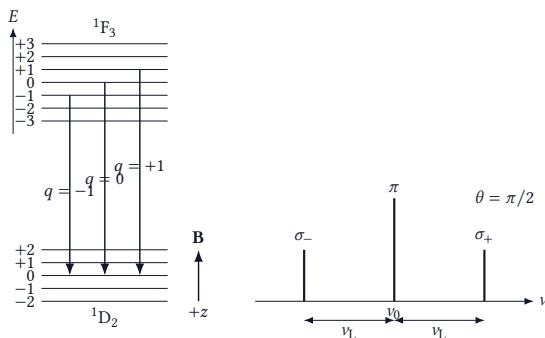
this becomes

$$\nu = \nu_0 + q\nu_L, \quad q = 0, \pm 1.$$

Although there are fifteen allowed magnetic-sublevel transitions, all transitions with the same q have the same frequency. They therefore form only three resolved spectral components:

q	allowed (M_u, M_l)	frequency	component
-1	$(-3, -2), (-2, -1), (-1, 0), (0, 1), (1, 2)$	$\nu_0 - \nu_L$	σ_-
0	$(-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2)$	ν_0	π
+1	$(-1, -2), (0, -1), (1, 0), (2, 1), (3, 2)$	$\nu_0 + \nu_L$	σ_+

The sublevel splitting, representative transitions, and the spectrum for observation perpendicular to $\mathbf{B}$ are shown below. The central line is drawn with twice the height of either side line because the transverse intensity ratio is $2 : 1 : 1$.



For an equally populated upper level, rotational symmetry and the Wigner-Eckart theorem imply equal total angle-integrated strengths for the three groups $q = -1, 0, +1$. Their observed intensities nevertheless depend on the angle θ between the line of sight and $\mathbf{B}$. The angular factors are proportional to

$$I_\pi(\theta) \propto \sin^2 \theta, \quad I_{\sigma_\pm}(\theta) \propto \frac{1 + \cos^2 \theta}{2}.$$

Consequently, for transverse observation, $\theta = \pi/2$,

$$I_\pi : I_{\sigma_+} : I_{\sigma_-} = 2 : 1 : 1.$$

The π component is then linearly polarized parallel to $\mathbf{B}$, whereas the two σ components are linearly polarized perpendicular to $\mathbf{B}$.

For longitudinal observation, $\theta = 0$,

$$I_\pi = 0, \quad I_{\sigma_+} = I_{\sigma_-},$$

and the two σ components are oppositely circularly polarized. Their handedness is interchanged if the direction of observation is reversed.

As $B \rightarrow 0$, $\nu_L \rightarrow 0$, so all three components merge continuously into the original line at ν_0 , as required.

CONCLUSION:

The $^1F_3 - ^1D_2$ transition gives a normal Zeeman triplet because both singlet levels have

$$g_u = g_l = 1.$$

With $\mathbf{B} = B\hat{z}$, $B > 0$, and $q = M_u - M_l$, the three components are

$$\begin{aligned} \nu_{\sigma_-} &= \nu_0 - \frac{\mu_B B}{h} \\ \nu_\pi &= \nu_0 \\ \nu_{\sigma_+} &= \nu_0 + \frac{\mu_B B}{h}. \end{aligned}$$

Here $\mu_B B/h$ is in hertz. The adjacent frequency separation is

$$\Delta\nu = \frac{\mu_B B}{h},$$

and the separation of the two σ components is

$$\nu_{\sigma_+} - \nu_{\sigma_-} = \frac{2\mu_B B}{h}.$$

Thus σ_- is displaced toward lower frequency and longer wavelength, while σ_+ is displaced toward higher frequency and shorter wavelength. For transverse observation the polarization directions are

$$\pi \parallel \mathbf{B}, \quad \sigma_\pm \perp \mathbf{B},$$

with intensity ratio

$$I_\pi : I_{\sigma_+} : I_{\sigma_-} = 2 : 1 : 1.$$

For longitudinal observation the π component is absent and the two σ components are oppositely circularly polarized. These results apply in the weak-field Zeeman regime in which J remains a good quantum number.

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Q7. $n_1 = 6$, $l_1 = 3$, $s_1 = \frac{1}{2}$, $n_2 = 5$, $l_2 = 1$, $s_2 = \frac{1}{2}$. The quantum numbers of two electrons in a two-valence electron atom are $n_1 = 6$, $l_1 = 3$, $s_1 = \frac{1}{2}$, $n_2 = 5$, $l_2 = 1$, $s_2 = \frac{1}{2}$. Assuming L - S coupling, find the possible values of L and J .

INTRODUCTION:

In L - S coupling, the individual orbital angular momenta first combine to form

$$\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2,$$

and the two electron spins combine to form

$$\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2.$$

The resultant orbital and spin angular momenta then combine according to

$$\mathbf{J} = \mathbf{L} + \mathbf{S}.$$

Here L , S , and J denote the corresponding non-negative angular-momentum quantum numbers, with magnitudes

$$|\mathbf{L}| = \hbar\sqrt{L(L+1)}$$

$$|\mathbf{S}| = \hbar\sqrt{S(S+1)}$$

$$|\mathbf{J}| = \hbar\sqrt{J(J+1)}.$$

An arbitrary fixed z -axis may be used as the quantization axis; the allowed resultant quantum numbers do not depend on this choice. The electrons occupy the distinct orbitals $6f$ and $5p$, so they are non-equivalent electrons. Consequently, no additional Pauli exclusion restriction removes any of the angular-momentum couplings obtained below.

SOLUTION:

For the addition of two angular momenta l_1 and l_2 , the allowed values of the resultant orbital quantum number are

$$L = |l_1 - l_2|, |l_1 - l_2| + 1, \dots, l_1 + l_2.$$

Substituting $l_1 = 3$ and $l_2 = 1$,

$$L = |3 - 1|, |3 - 1| + 1, \dots, 3 + 1,$$

and hence

$$L = 2, 3, 4.$$

These values are conventionally denoted by the spectroscopic letters

$$L = 2 \leftrightarrow D, \quad L = 3 \leftrightarrow F, \quad L = 4 \leftrightarrow G.$$

The two spin quantum numbers are $s_1 = s_2 = \frac{1}{2}$. Their resultant spin is therefore

$$S = |s_1 - s_2|, \dots, s_1 + s_2 = \left| \frac{1}{2} - \frac{1}{2} \right|, \dots, \left(\frac{1}{2} + \frac{1}{2} \right),$$

so that

$$S = 0, 1.$$

Thus both singlet states, with multiplicity $2S + 1 = 1$, and triplet states, with multiplicity $2S + 1 = 3$, are possible.

For each pair (L, S) , the total angular-momentum quantum number satisfies

$$J = |L - S|, |L - S| + 1, \dots, L + S.$$

For the singlet states, $S = 0$, so

$$J = |L - 0| = L.$$

Therefore,

L	S	J
2	0	2
3	0	3
4	0	4

and the corresponding singlet terms are

$$^1D_2, \quad ^1F_3, \quad ^1G_4.$$

For the triplet states, $S = 1$, so for every allowed $L \geq 1$,

$$J = L - 1, L, L + 1.$$

Consequently,

L	S	J
2	1	1, 2, 3
3	1	2, 3, 4
4	1	3, 4, 5

and the corresponding triplet terms are

$$^3D_1, ^3D_2, ^3D_3,$$

$$^3F_2, ^3F_3, ^3F_4,$$

$$^3G_3, ^3G_4, ^3G_5.$$

The extreme values provide a useful angular-momentum check: the largest possible total angular momentum is

$$J_{\max} = l_1 + l_2 + s_1 + s_2 = 3 + 1 + \frac{1}{2} + \frac{1}{2} = 5,$$

which agrees with the largest value $J = 5$ obtained for $L = 4, S = 1$. The smallest value is $J = 1$, obtained from the antiparallel coupling of $L = 2$ and $S = 1$.

CONCLUSION:

The allowed resultant orbital angular-momentum quantum numbers are

$$L = 2, 3, 4.$$

Including both allowed total-spin values $S = 0$ and $S = 1$, the possible J -values associated with each L are

$$\begin{array}{llll} L = 2 : & J = 2 & (S = 0), & J = 1, 2, 3 & (S = 1), \\ L = 3 : & J = 3 & (S = 0), & J = 2, 3, 4 & (S = 1), \\ L = 4 : & J = 4 & (S = 0), & J = 3, 4, 5 & (S = 1). \end{array}$$

Thus the distinct possible numerical values of the total angular-momentum quantum number are

$$J = 1, 2, 3, 4, 5,$$

with the allowed L - S - J associations specified above. These results apply in the assumed L - S coupling regime for the non-equivalent $6f$ and $5p$ electrons.

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Q8. Find out the difference in frequencies of Lyman-alpha line in hydrogen and deuterium atoms.

INTRODUCTION:

The Lyman- α line is produced by the transition

$$n = 2 \longrightarrow n = 1.$$

Hydrogen and deuterium have identical nuclear charge $+e$, but their nuclei have different masses. Consequently, their electron-nucleus reduced masses, and hence their Rydberg constants and transition frequencies, are slightly different.

Let m_e , m_p , and m_d denote the electron, proton, and deuteron masses, respectively. The reduced mass of an electron bound to a nucleus of mass M is

$$\mu_M = \frac{m_e M}{m_e + M}.$$

Nuclear recoil is included through this reduced mass. Fine structure, Lamb shift, and nuclear-size corrections are neglected.

SOLUTION:

For a hydrogen-like atom of nuclear charge $Z = 1$, the energy of the n th level is

$$E_n = -hcR_\infty \frac{\mu_M}{m_e} \frac{1}{n^2},$$

where R_∞ is the Rydberg constant for an infinitely massive nucleus.

Thus, for the Lyman- α transition,

$$h\nu_M = E_2 - E_1 = hcR_\infty \frac{\mu_M}{m_e} \left(1 - \frac{1}{2^2}\right),$$

so that

$$\nu_M = \frac{3}{4} cR_\infty \frac{\mu_M}{m_e}.$$

For ordinary hydrogen,

$$\frac{\mu_H}{m_e} = \frac{m_p}{m_e + m_p} = \frac{1}{1 + m_e/m_p},$$

whereas for deuterium,

$$\frac{\mu_D}{m_e} = \frac{m_d}{m_e + m_d} = \frac{1}{1 + m_e/m_d}.$$

Since $m_d > m_p$, one has $\mu_D > \mu_H$, and therefore the Lyman- α frequency of deuterium is higher. The frequency difference is

$$\Delta\nu \equiv \nu_D - \nu_H = \frac{3}{4} cR_\infty \left[\frac{1}{1 + m_e/m_d} - \frac{1}{1 + m_e/m_p} \right].$$

Using

$$\begin{aligned} \frac{m_p}{m_e} &\approx 1836.15 \\ \frac{m_d}{m_e} &\approx 3670.48 \end{aligned}$$

$$cR_\infty \approx 3.28984 \times 10^{15} \text{ s}^{-1}$$

gives

$$\Delta\nu = \frac{3}{4} (3.28984 \times 10^{15}) \left[\frac{1}{1 + 1/3670.48} - \frac{1}{1 + 1/1836.15} \right] \text{ Hz}.$$

The dimensionless reduced-mass difference is approximately

$$\frac{1}{1 + 1/3670.48} - \frac{1}{1 + 1/1836.15} \approx 2.72 \times 10^{-4}.$$

Therefore,

$$\Delta\nu \approx (2.46738 \times 10^{15})(2.72 \times 10^{-4}) \text{ Hz} \approx 6.7 \times 10^{11} \text{ Hz}.$$

The positive sign is physically consistent: the heavier deuteron recoils less than the proton, giving deuterium a larger reduced mass, a larger binding-energy separation, and hence a higher transition frequency.

CONCLUSION:

The difference between the Lyman- α frequencies of deuterium and hydrogen is

$$\nu_D - \nu_H \approx 6.7 \times 10^{11} \text{ Hz}.$$

Thus, the deuterium Lyman- α line has the higher frequency and, equivalently, the shorter wavelength.

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Q9. Why were silver atoms used in Stern-Gerlach experiment? Also, write the importance of this experiment.

INTRODUCTION:

The Stern-Gerlach experiment examines the behaviour of a beam of neutral atoms in a non-uniform magnetic field. If the field is taken along the z -direction, the force on an atom having magnetic-moment component μ_z is

$$F_z = \mu_z \frac{\partial B_z}{\partial z}.$$

Thus, different allowed values of μ_z produce different transverse deflections. Silver is especially suitable because its electronic structure makes the observed magnetic moment simple to interpret.

SOLUTION:

The electronic configuration of silver, $Z = 47$, is

$$\text{Ag} : [\text{Kr}] 4d^{10}5s^1.$$

The filled inner shells, including the $4d^{10}$ subshell, have paired electrons. Their orbital and spin angular momenta cancel, so they make no resultant contribution:

$$L_{\text{closed}} = 0, \quad S_{\text{closed}} = 0.$$

The single outer $5s$ electron has

$$l = 0, \quad s = \frac{1}{2}, \quad j = \frac{1}{2},$$

and hence the ground state is

$$^2S_{1/2}.$$

Since $l = 0$, the valence electron has no orbital angular momentum. The magnetic moment of the atom is therefore essentially the spin magnetic moment of this one unpaired electron. Its component along the magnetic field can take only two values because

$$m_j = \pm \frac{1}{2}.$$

Using

$$\mu_z = -g_J \mu_B m_j, \quad g_J \approx 2,$$

one obtains

$$\mu_z \approx \mp \mu_B,$$

where

$$\mu_B = \frac{e\hbar}{2m_e}$$

is the Bohr magneton. Consequently,

$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

has two opposite values, and the silver-atom beam separates into two components.

Silver atoms also have practical experimental advantages: a well-collimated neutral atomic beam can be produced by heating silver in an oven, and the atoms can be detected from the visible deposit formed on a collecting plate. Their neutrality prevents the ordinary Lorentz force on electric charge from obscuring the magnetic-moment deflection.

Classically, atomic magnetic moments could have arbitrary orientations. A classical beam would therefore be expected to form a continuous distribution of deflections. Instead, Stern and Gerlach observed two sharply separated components. The experiment consequently established the following points:

1. It provided direct evidence for *space quantization*: the component of atomic angular momentum, and hence of magnetic moment, along a chosen axis can assume only discrete values.
 2. The twofold splitting of the silver beam was later understood as evidence for the two allowed projections of electron spin,
- $$m_s = +\frac{1}{2}, \quad m_s = -\frac{1}{2}.$$
3. It demonstrated that measurement of an angular-momentum component gives discrete quantum outcomes rather than a continuous range of classical values.
 4. It supplied important experimental support for quantum mechanics and for the association between angular momentum and magnetic moment.
 5. It introduced the principle of separating quantum states by their magnetic moments, which underlies atomic-beam magnetic-resonance methods and later techniques for manipulating and measuring spin states.

Historically, the experiment was initially interpreted in terms of orbital space quantization because electron spin had not yet been proposed. The modern interpretation of the two silver components is based on the spin of the single $5s$ electron.

CONCLUSION:

Silver was used because its configuration $[\text{Kr}] 4d^{10}5s^1$ contains closed shells with zero resultant angular momentum and one unpaired s -electron with $l = 0$ and $j = \frac{1}{2}$. Its magnetic moment is therefore essentially a single-electron spin moment with two allowed field

components, approximately $\mu_z = \pm\mu_B$, producing two opposite deflections in a non-uniform magnetic field. The experiment established discrete space quantization, gave decisive evidence for the two-valued projection of electron spin, contradicted the classical prediction of continuous orientations, and became a foundational demonstration of quantum-state measurement and magnetic-state separation.

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Q10. Write the electronic configurations for carbon (C), nitrogen (N) and oxygen (O) atoms, and then derive their ground states.

INTRODUCTION:

The atomic numbers are $Z_C = 6$, $Z_N = 7$, and $Z_O = 8$. Their electronic configurations follow from the Aufbau principle, the Pauli exclusion principle, and Hund's rules.

For these atoms, the closed subshells $1s^2 2s^2$ have

$$L = 0, \quad S = 0,$$

and therefore do not affect the resultant term symbol. It is sufficient to couple the electrons in the open $2p^n$ subshell. For a p -electron,

$$l = 1, \quad m_l = +1, 0, -1, \quad s = \frac{1}{2}.$$

In Russell–Saunders coupling, an atomic level is denoted by

$$^{2S+1}L_J,$$

where S is the total spin, $L = 0, 1, 2, \dots$ is represented by $S, P, D, \dots$, and

$$J = |L - S|, |L - S| + 1, \dots, L + S.$$

Odd parity is indicated by a superscript $^\circ$.

Hund's rules select the ground level as follows: maximize S ; for that value of S , maximize L ; and then choose the minimum J for a less-than-half-filled subshell and the maximum J for a more-than-half-filled subshell. The $2p$ subshell is half-filled at $2p^3$.

SOLUTION:

- (a) For carbon, $Z = 6$, so the electronic configuration is

$$\boxed{\text{C} : 1s^2 2s^2 2p^2}.$$

The two $2p$ -electrons occupy different orbitals with parallel spins:

$$\begin{array}{c|ccc} m_l & +1 & 0 & -1 \\ \hline 2p^2 & \uparrow & \uparrow & \end{array}$$

Thus

$$M_S = \frac{1}{2} + \frac{1}{2} = 1,$$

which is the maximum possible value and therefore gives

$$S = 1, \quad 2S + 1 = 3.$$

For two parallel-spin electrons, the Pauli principle requires different m_l values. The largest possible orbital projection is consequently

$$M_L = 1 + 0 = 1,$$

so the corresponding maximum- M_L state has

$$L = 1 \quad (P).$$

Hence Hund's first two rules select the term

$$^3P.$$

Equivalently, the complete allowed terms of a p^2 configuration are

$$^3P, \quad ^1D, \quad ^1S,$$

and the triplet term lies lowest because it has the largest spin.

For $L = 1$ and $S = 1$,

$$J = 0, 1, 2.$$

Since $2p^2$ is less than half-filled, Hund's third rule selects the smallest value,

$$J = |L - S| = 0.$$

The parity is even because

$$(-1)^{2l} = (-1)^2 = +1.$$

Therefore the carbon ground state is

$$\boxed{\text{C} : 1s^2 2s^2 2p^2 \ ^3P_0}.$$

- (b) For nitrogen, $Z = 7$, so the electronic configuration is

$$\boxed{\text{N} : 1s^2 2s^2 2p^3}.$$

To maximize the total spin, the three $2p$ -electrons occupy all three p -orbitals with parallel spins:

$$\begin{array}{c|ccc} m_l & +1 & 0 & -1 \\ \hline 2p^3 & \uparrow & \uparrow & \uparrow \end{array}$$

Therefore

$$M_S = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2},$$

and hence

$$S = \frac{3}{2}, \quad 2S + 1 = 4.$$

The orbital projection of this unique maximum-spin arrangement is

$$M_L = (+1) + 0 + (-1) = 0.$$

Since all three available m_l orbitals must be singly occupied in the maximum-spin state, no state with $S = 3/2$ can have $M_L \neq 0$. Thus

$$L = 0 \quad (S).$$

Hund's rules therefore select

$$^4S.$$

The other terms of p^3 , such as ^2D and ^2P , have smaller spin and lie higher.

For $L = 0$ and $S = 3/2$, only one value of J is possible:

$$J = \frac{3}{2}.$$

The parity is odd because

$$(-1)^{3l} = (-1)^3 = -1.$$

Consequently the nitrogen ground state is

$$\boxed{\text{N} : 1s^2 2s^2 2p^3 \ ^4S_{3/2}^o}.$$

- (c) For oxygen, $Z = 8$, so the electronic configuration is

$$\boxed{\text{O} : 1s^2 2s^2 2p^4}.$$

Hund's first rule gives three parallel-spin electrons in the three different p -orbitals; the fourth electron must pair with opposite spin:

$$\frac{m_l}{2p^4} \begin{array}{|c|c|c|} \hline +1 & 0 & -1 \\ \hline \uparrow\downarrow & \uparrow & \uparrow \\ \hline \end{array}$$

For this maximum-spin arrangement,

$$M_S = \left(\frac{1}{2} - \frac{1}{2}\right) + \frac{1}{2} + \frac{1}{2} = 1,$$

so

$$S = 1, \quad 2S + 1 = 3.$$

Choosing the paired orbital at $m_l = +1$ gives the largest orbital projection compatible with $S = 1$:

$$M_L = (+1) + (+1) + 0 + (-1) = 1.$$

Therefore

$$L = 1 \quad (P),$$

and Hund's first two rules select the term

$$^3P.$$

The same result follows from the hole description: a p^4 configuration contains two holes in the otherwise closed p^6 subshell and therefore has the same allowed L and S terms as p^2 :

$$^3P, \quad ^1D, \quad ^1S.$$

For $L = 1$ and $S = 1$,

$$J = 0, 1, 2.$$

Since $2p^4$ is more than half-filled, Hund's third rule selects the largest value,

$$J = L + S = 2.$$

The parity is even:

$$(-1)^{4l} = (-1)^4 = +1.$$

Hence the oxygen ground state is

$$\boxed{\text{O} : 1s^2 2s^2 2p^4 \ ^3P_2}.$$

The contrast between carbon and oxygen provides a useful check: p^2 and p^4 have the same lowest term ^3P , but their ground J -values are opposite members of the multiplet because p^2 is less than half-filled whereas p^4 is more than half-filled.

CONCLUSION:

Under Russell–Saunders coupling for isolated light atoms, the electronic configurations and ground-state term symbols are

C : $1s^2 2s^2 2p^2$,	ground state $^3P_0$,
N : $1s^2 2s^2 2p^3$,	ground state $^4S_{3/2}^o$,
O : $1s^2 2s^2 2p^4$,	ground state $^3P_2$.

Thus carbon and oxygen possess triplet- P ground terms but different ground J -levels, while the exactly half-filled $2p^3$ subshell of nitrogen produces a spin-aligned, orbitally non-degenerate quartet- S ground state of odd parity.



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